

# **Introduction to hadronic collisions: theoretical concepts and practical tools for the LHC**

## **Lecture 2**

*Università Roma I, La Sapienza  
12-15 Febbraio 2007*

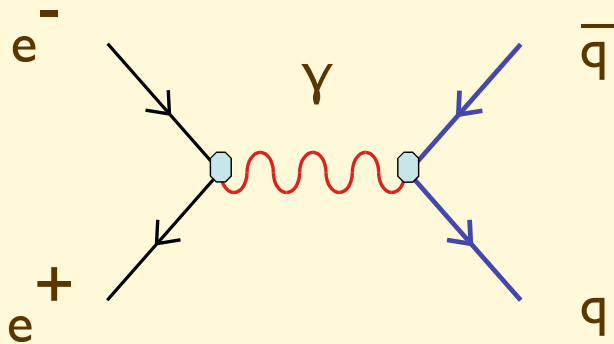
**Michelangelo L. Mangano**  
TH Unit, Physics Dept, CERN  
[michelangelo.mangano@cern.ch](mailto:michelangelo.mangano@cern.ch)

# Evolution of hadronic final states

Asymptotic freedom implies that at  $E_{\text{CM}} \gg 1 \text{ GeV}$

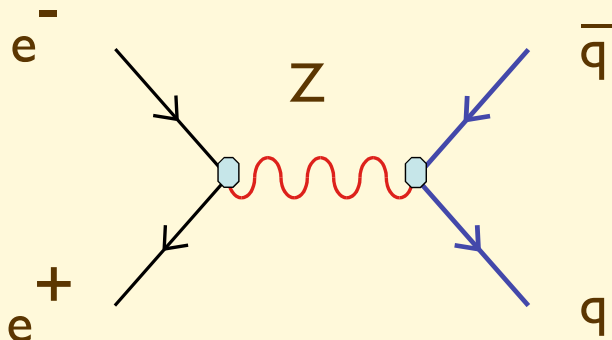
$$\sigma(e^+ e^- \rightarrow \text{hadrons}) \longleftrightarrow \sigma(e^+ e^- \rightarrow \text{quarks/gluons})$$

At the Leading Order (LO) in PT:



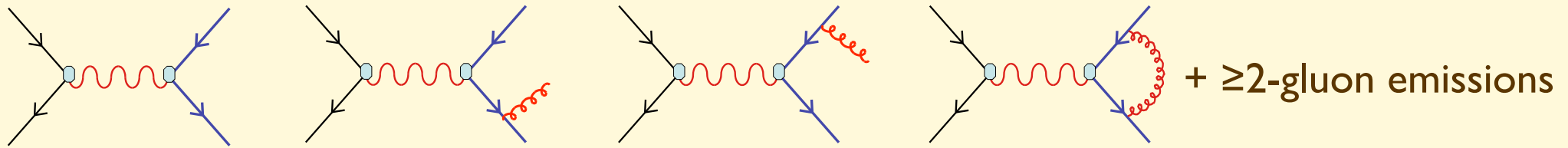
$$\sigma_0(e^+ e^- \rightarrow q \bar{q}) = \frac{4\pi\alpha^2}{9s} N_c \sum_{f=u,d,\dots} e_{q_f}^2$$

$$\frac{\sigma_0(e^+ e^- \rightarrow q \bar{q})}{\sigma_0(e^+ e^- \rightarrow \mu^+ \mu^-)} = N_c \sum_{f=u,d,\dots} e_{q_f}^2$$



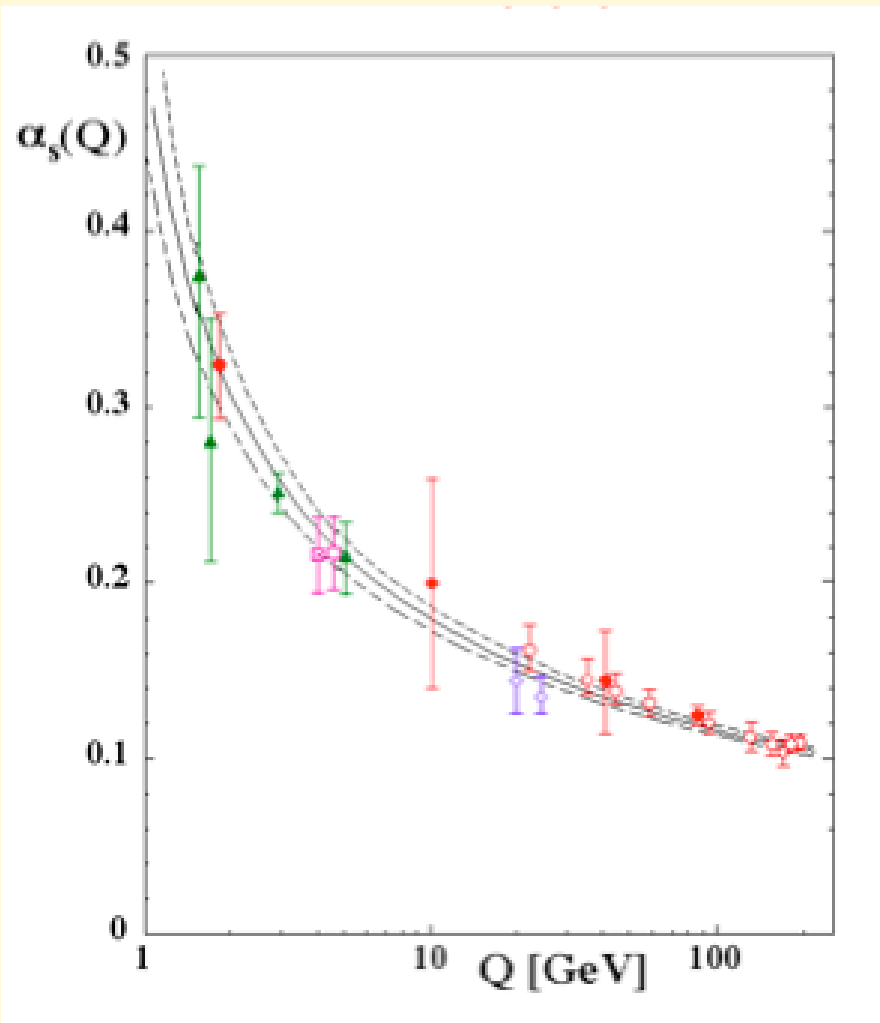
$$\frac{\sigma_0(e^+ e^- \rightarrow Z \rightarrow q \bar{q})}{\sigma_0(e^+ e^- \rightarrow Z \rightarrow \mu^+ \mu^-)} = N_c \frac{\sum_{f=u,d,\dots} (v_{q_f}^2 + a_{q_f}^2)}{(v_\mu^2 + a_\mu^2)}$$

## Adding higher-order perturbative terms:



$$\sigma_1(e^+e^- \rightarrow q\bar{q}(g)) = \sigma_0(e^+e^- \rightarrow q\bar{q}) \left( 1 + \frac{\alpha_s(E_{CM})}{\pi} + O(\alpha_s^2) \right)$$

O(3%) at  $M_Z$



Excellent agreement with data,

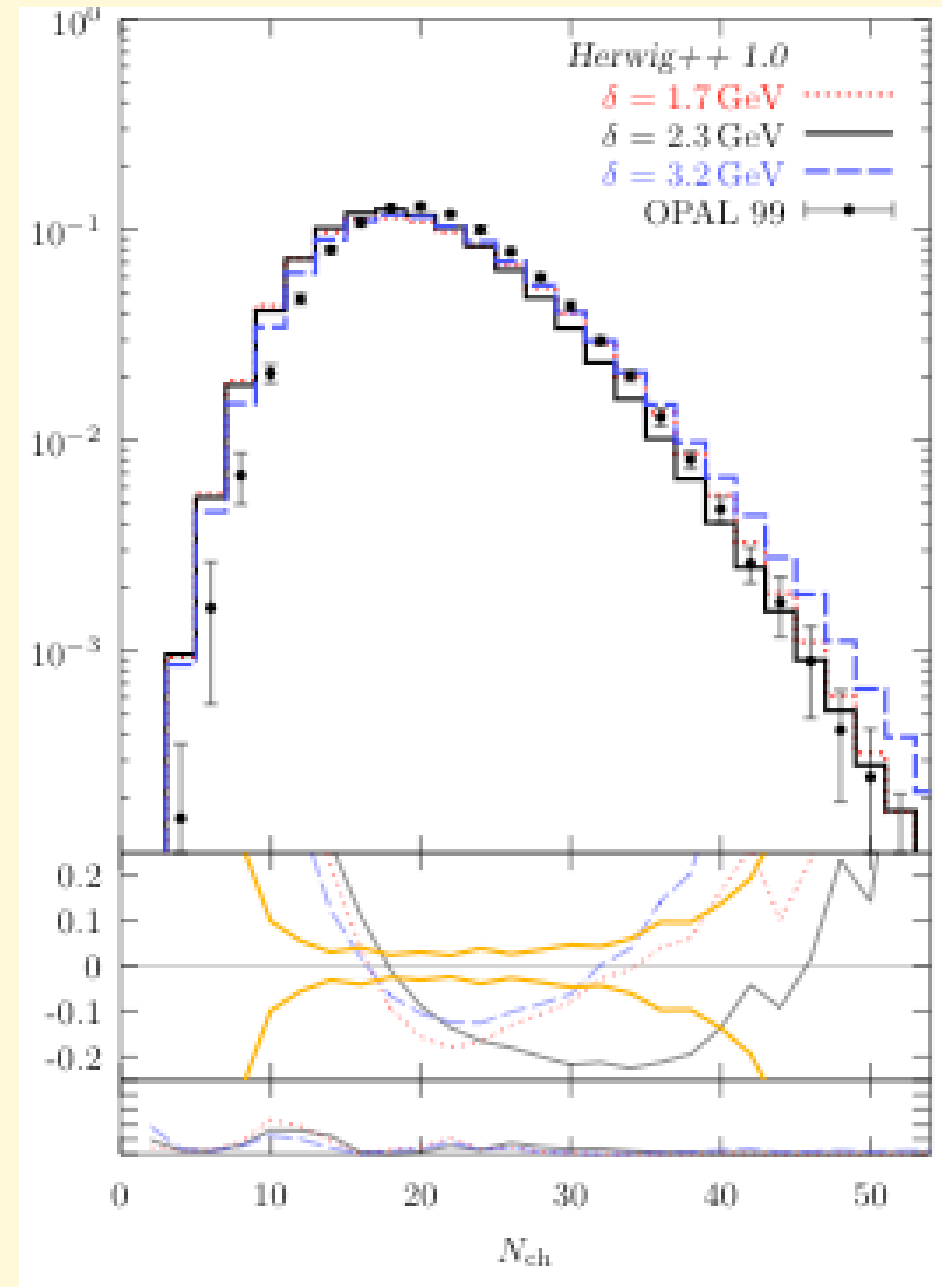
**provided  $N_c=3$**

Extraction of  $\alpha_s$  consistent with the  $Q$  evolution predicted by QCD

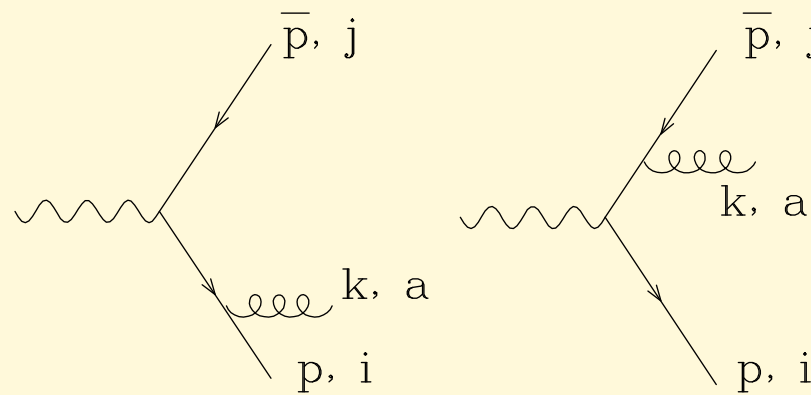
Experimentally, the final states contain a large number of particles, not the 2 or 3 which apparently saturate the perturbative cross-section.

Experimental  
multiplicity  
distribution

$$\langle n_{\text{charged}} \rangle = 20.9$$



# Soft gluon emission



$$\begin{aligned}
 A &= \bar{u}(p)\epsilon(k)(ig)\frac{-i}{\not{p}+\not{k}}\Gamma^\mu v(\bar{p})\lambda_{ij}^a + \bar{u}(p)\Gamma^\mu\frac{i}{\not{p}+\not{k}}(ig)\epsilon(k)v(\bar{p})\lambda_{ij}^a \\
 &= \left[ \frac{g}{2p\cdot k}\bar{u}(p)\epsilon(k)(\not{p}+\not{k})\Gamma^\mu v(\bar{p}) - \frac{g}{2\bar{p}\cdot k}\bar{u}(p)\Gamma^\mu(\not{p}+\not{k})\epsilon(k)v(\bar{p}) \right] \lambda_{ij}^a
 \end{aligned}$$

$p\cdot k = p_0 k_0 (1-\cos\theta) \Rightarrow$  singularities for collinear ( $\cos\theta \rightarrow 1$ ) or soft ( $k_0 \rightarrow 0$ ) emission

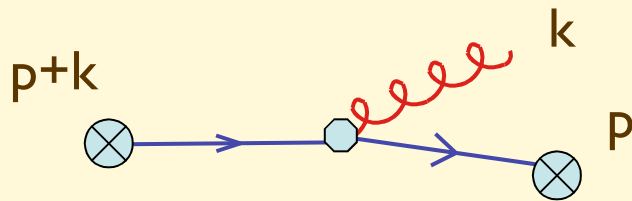
**Collinear emission** does not alter the global structure of the final state, since it preserves its “pencil-like-ness”. **Soft emission** at large angle, however, could spoil the structure, and leads to strong interferences between emissions from different legs. So soft emission needs to be studied in more detail.

In the soft ( $k_0 \rightarrow 0$ ) limit the amplitude simplifies and factorizes as follows:

$$A_{soft} = g\lambda_{ij}^a \left( \frac{p\cdot\epsilon}{p\cdot k} - \frac{\bar{p}\cdot\epsilon}{\bar{p}\cdot k} \right) A_{Born}$$

**Factorization:** it is the expression of the independence of long-wavelength (soft) emission on the nature of the hard (short-distance) process.

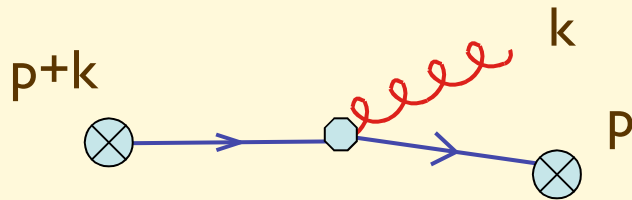
# Another simple derivation of soft-gluon emission rules



charge current of  
a free fermion

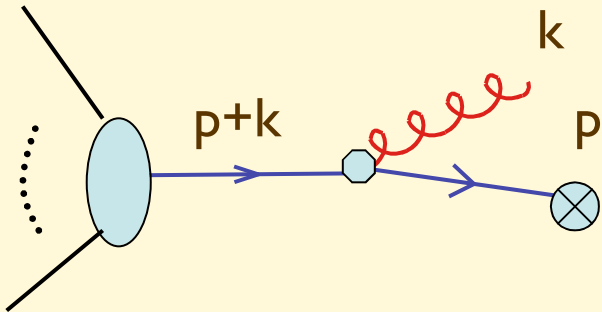
$$\bar{\Psi}(p) \gamma_{\mu} \Psi(p+k) \varepsilon^{\mu}(k) \xrightarrow{k \rightarrow 0} \bar{\Psi}(p) \gamma_{\mu} \Psi(p) \varepsilon^{\mu}(k) = 2p \cdot \varepsilon$$

# Another simple derivation of soft-gluon emission rules



charge current of  
a free fermion

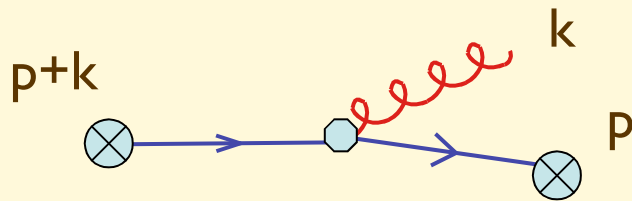
$$\bar{\Psi}(p) \gamma_{\mu} \Psi(p+k) \varepsilon^{\mu}(k) \xrightarrow{k \rightarrow 0} \bar{\Psi}(p) \gamma_{\mu} \Psi(p) \varepsilon^{\mu}(k) = 2p \cdot \varepsilon$$



$$\frac{1}{\not{p} + \not{k}} \gamma_{\mu} \Psi(p+k) \varepsilon^{\mu}(k) \xrightarrow{k \rightarrow 0}$$

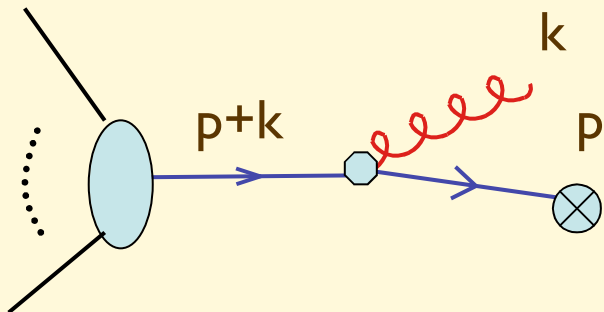
$$\frac{1}{2p \cdot k} \not{p} \gamma_{\mu} \Psi(p) \varepsilon^{\mu}(k) = \frac{p \cdot \varepsilon}{p \cdot k}$$

# Another simple derivation of soft-gluon emission rules



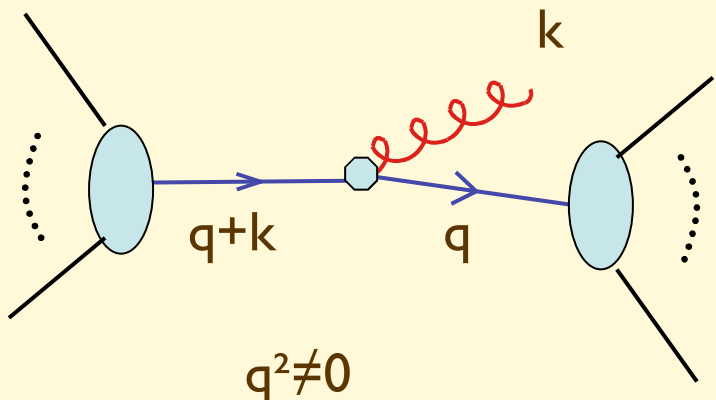
charge current of  
a free fermion

$$\bar{\Psi}(p) \gamma_{\mu} \Psi(p+k) \varepsilon^{\mu}(k) \xrightarrow{k \rightarrow 0} \bar{\Psi}(p) \gamma_{\mu} \Psi(p) \varepsilon^{\mu}(k) = 2p \cdot \varepsilon$$



$$\frac{1}{\not{p} + \not{k}} \gamma_{\mu} \Psi(p+k) \varepsilon^{\mu}(k) \xrightarrow{k \rightarrow 0}$$

$$\frac{1}{2p \cdot k} \not{p} \gamma_{\mu} \Psi(p) \varepsilon^{\mu}(k) = \frac{p \cdot \varepsilon}{p \cdot k}$$

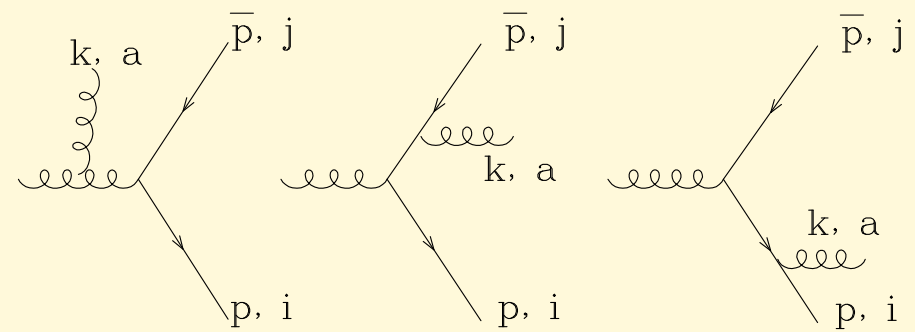


$$\frac{1}{\not{q} + \not{k}} \gamma_{\mu} \frac{1}{\not{q}} \varepsilon^{\mu}(k) \xrightarrow{q^2 \neq 0, k \rightarrow 0} \frac{1}{q^2} \not{q} \gamma_{\mu} \not{q} \frac{1}{q^2} \varepsilon^{\mu}(k)$$

=> finite

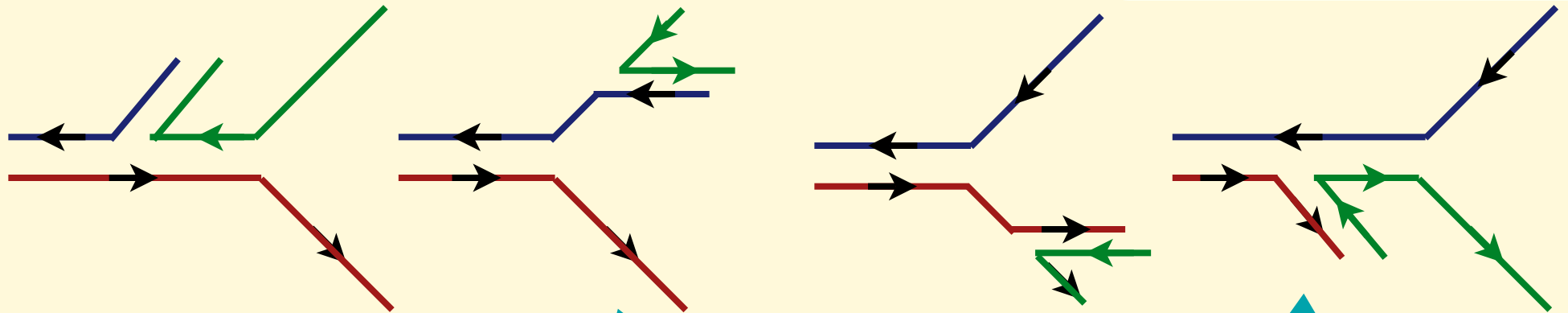
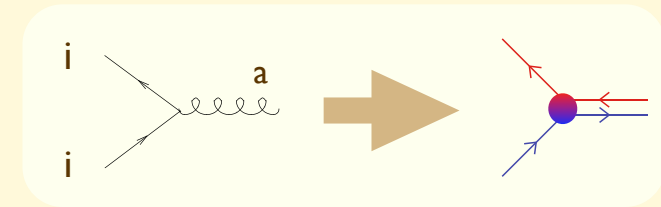


Similar, but more structured, result in the case of a fully coloured process:



$$A_{soft} = g (\lambda^a \lambda^b)_{ij} \left[ \frac{Q\varepsilon}{Qk} - \frac{\bar{p}\varepsilon}{\bar{p}k} \right] + g (\lambda^b \lambda^a)_{ij} \left[ \frac{p\varepsilon}{pk} - \frac{Q\varepsilon}{Qk} \right]$$

The four terms correspond to the two possible ways colour can flow, and to the two possible emissions for each colour flow:



$$A_{soft} = g (\lambda^a \lambda^b)_{ij} \left[ \frac{Q\varepsilon}{Qk} - \frac{\bar{p}\varepsilon}{\bar{p}k} \right] + g (\lambda^b \lambda^a)_{ij} \left[ \frac{p\varepsilon}{pk} - \frac{Q\varepsilon}{Qk} \right]$$

The interference between the two colour structures

$$\left[ \text{Diagram 1} + \text{Diagram 2} \right] \propto (\lambda^a \lambda^b)_{ij} \quad \left[ \text{Diagram 3} + \text{Diagram 4} \right] \propto (\lambda^b \lambda^a)_{ij}$$

is suppressed by  $1/N_c^2$ :

$$\sum_{a,b,i,j} |(\lambda^a \lambda^b)_{ij}|^2 = \sum_{a,b} \text{tr} (\lambda^a \lambda^b \lambda^b \lambda^a) = \frac{N^2 - 1}{2} C_F = O(N^3)$$

$$\sum_{a,b,i,j} (\lambda^a \lambda^b)_{ij} [(\lambda^b \lambda^a)_{ij}]^* = \sum_{a,b} \text{tr} (\lambda^a \lambda^b \lambda^a \lambda^b) = \frac{N^2 - 1}{2} \underbrace{\left( C_F - \frac{C_A}{2} \right)}_{-\frac{1}{2N}} = O(N)$$

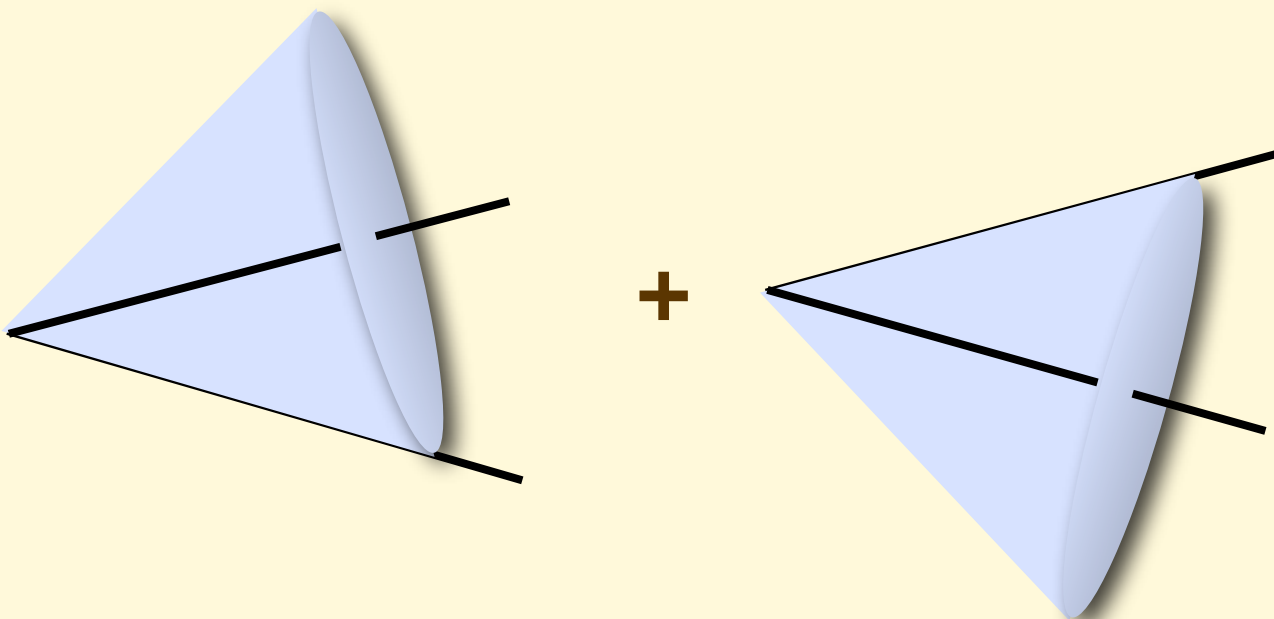
As a result, the emission of a soft gluon can be described, to the leading order in  $1/N_c^2$ , as the incoherent sum of the emission from the two colour currents

What about the interference between the two diagrams corresponding to the same colour flow? ➡

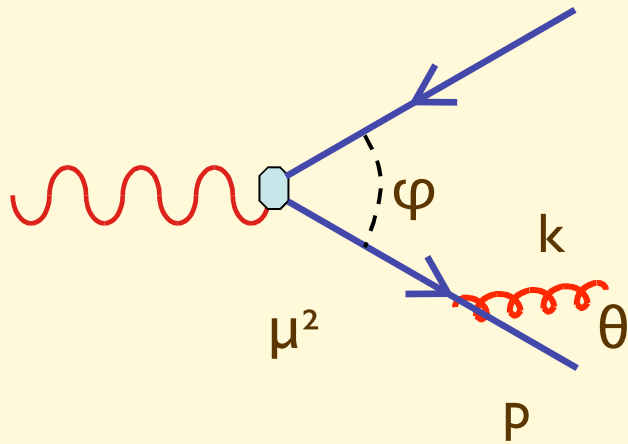
# Angular ordering

$$\left| \begin{array}{c} \text{Diagram: A blue oval vertex with a wavy line to the left and two lines to the right, one of which has a wavy line branching off at an angle.} \end{array} \right|^2 = \left| \begin{array}{c} \text{Diagram: A vertex with a wavy line to the left and two lines to the right. One line branches off at angle } \varphi_1, \text{ and the other at angle } \varphi. \end{array} \right|^2 \Theta(\varphi - \varphi_1) + \left| \begin{array}{c} \text{Diagram: A vertex with a wavy line to the left and two lines to the right. One line branches off at angle } \varphi_2, \text{ and the other at angle } \varphi. \end{array} \right|^2 \Theta(\varphi - \varphi_2)$$

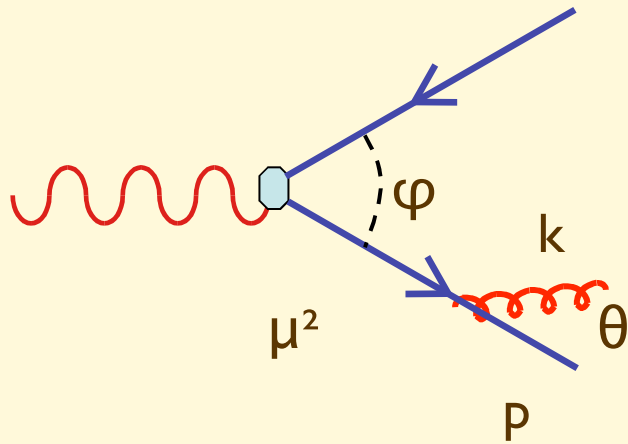
Radiation inside the cones is allowed, and described by the eikonal probability, radiation outside the cones is suppressed and averages to 0 when integrated over the full azimuth



# An intuitive explanation of angular ordering



# An intuitive explanation of angular ordering

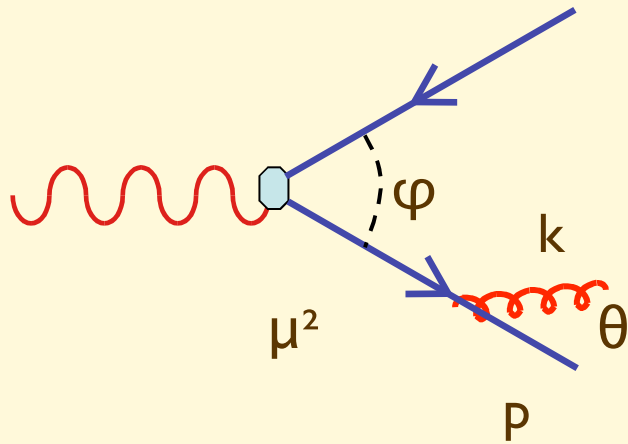


Lifetime of the virtual intermediate state:

$$\tau < \gamma/\mu = E/\mu^2 = 1/k_0 \theta^2 = 1/k_\perp \theta$$

$$\begin{aligned} \mu^2 &= (p+k)^2 = 2E k_0 (1-\cos\theta) \\ &\sim E k_0 \theta^2 \sim E k_\perp \theta \end{aligned}$$

# An intuitive explanation of angular ordering



Lifetime of the virtual intermediate state:

$$\tau < \gamma/\mu = E/\mu^2 = 1/k_0 \quad \theta^2 = 1/k_{\perp} \theta$$

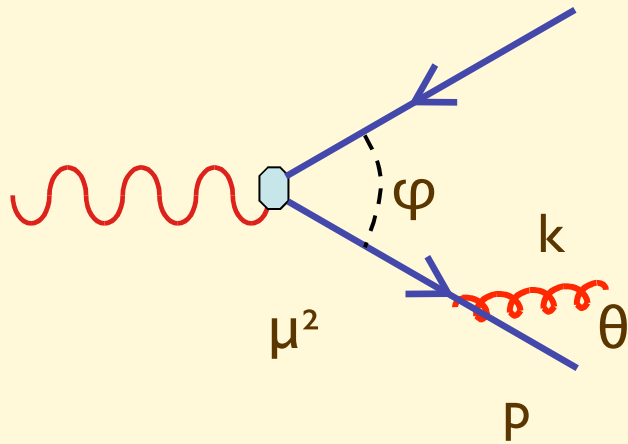
$$\begin{aligned} \mu^2 &= (p+k)^2 = 2E k_0 (1-\cos\theta) \\ &\sim E k_0 \theta^2 \sim E k_{\perp} \theta \end{aligned}$$

Distance between  $q$  and  $qbar$  after  $\tau$ :

$$d = \varphi\tau = (\varphi/\theta) 1/k_{\perp}$$

If the transverse wavelength of the emitted gluon is longer than the separation between  $q$  and  $qbar$ , the gluon emission is suppressed, because the  $q qbar$  system will appear as colour neutral ( $\Rightarrow$  dipole-like emission, suppressed)

# An intuitive explanation of angular ordering



Lifetime of the virtual intermediate state:

$$\tau < \gamma/\mu = E/\mu^2 = 1/k_0 \quad \theta^2 = 1/k_{\perp}^2$$

$$\begin{aligned} \mu^2 &= (p+k)^2 = 2E k_0 (1-\cos\theta) \\ &\sim E k_0 \theta^2 \sim E k_{\perp}^2 \end{aligned}$$

Distance between q and qbar after  $\tau$ :

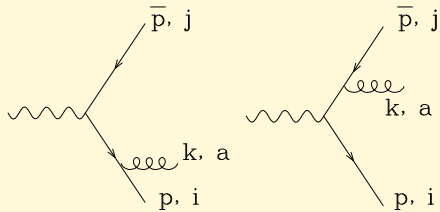
$$d = \varphi\tau = (\varphi/\theta) 1/k_{\perp}$$

If the transverse wavelength of the emitted gluon is longer than the separation between q and qbar, the gluon emission is suppressed, because the q qbar system will appear as colour neutral ( $\Rightarrow$  dipole-like emission, suppressed)

Therefore  $d > 1/k_{\perp}$ , which implies

$$\theta < \varphi$$

# The formal proof of angular ordering



$$d\sigma_g = \sum |A_{soft}|^2 \frac{d^3k}{(2\pi)^3 2k^0} \sum |A_0|^2 \frac{-2p^\mu \bar{p}^\nu}{(pk)(\bar{p}k)} g^2 \sum \epsilon_\mu \epsilon_\nu^* \frac{d^3k}{(2\pi)^3 2k^0}$$

$$= d\sigma_0 \frac{\alpha_s C_F}{\pi} \frac{dk^0}{k^0} \frac{d\phi}{2\pi} \frac{1 - \cos \theta_{ij}}{(1 - \cos \theta_{ik})(1 - \cos \theta_{jk})} d\cos \theta$$

You can easily prove that:

$$\frac{1 - \cos \theta_{ij}}{(1 - \cos \theta_{ik})(1 - \cos \theta_{jk})} = \frac{1}{2} \left[ \frac{\cos \theta_{jk} - \cos \theta_{ij}}{(1 - \cos \theta_{ik})(1 - \cos \theta_{jk})} + \frac{1}{1 - \cos \theta_{ik}} \right] + \frac{1}{2} [i \leftrightarrow j] \equiv W_{(i)} + W_{(j)}$$

where:

$$W_{(i)} \rightarrow \text{finite if } k \parallel j \text{ (} \cos \theta_{jk} \rightarrow 1 \text{)}$$

$$W_{(j)} \rightarrow \text{finite if } k \parallel i \text{ (} \cos \theta_{ik} \rightarrow 1 \text{)}$$

The probabilistic interpretation of  $W_{(i)}$  and  $W_{(j)}$  is a priori spoiled by their non-positivity. However, you can prove that after azimuthal averaging:

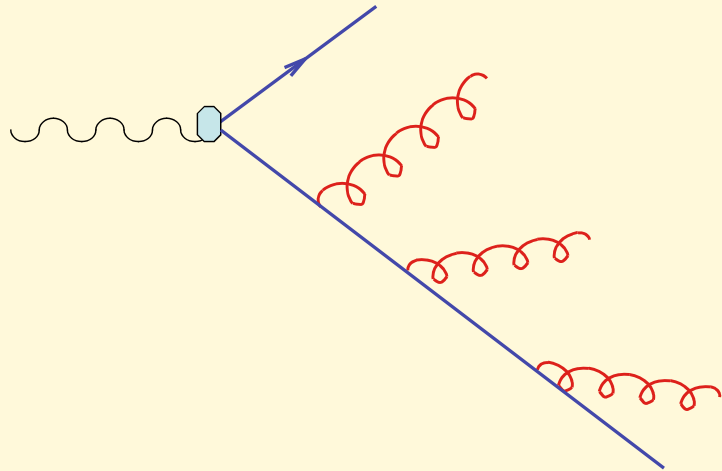
$$\left| \text{diagram} \right|^2 = \left| \text{diagram}_1 \right|^2 \Theta(\varphi - \varphi_1) + \left| \text{diagram}_2 \right|^2 \Theta(\varphi - \varphi_2)$$

$$\int \frac{d\phi}{2\pi} W_{(i)} = \frac{1}{1 - \cos \theta_{ik}} \text{ if } \theta_{ik} < \theta_{ij}, \quad 0 \text{ otherwise}$$

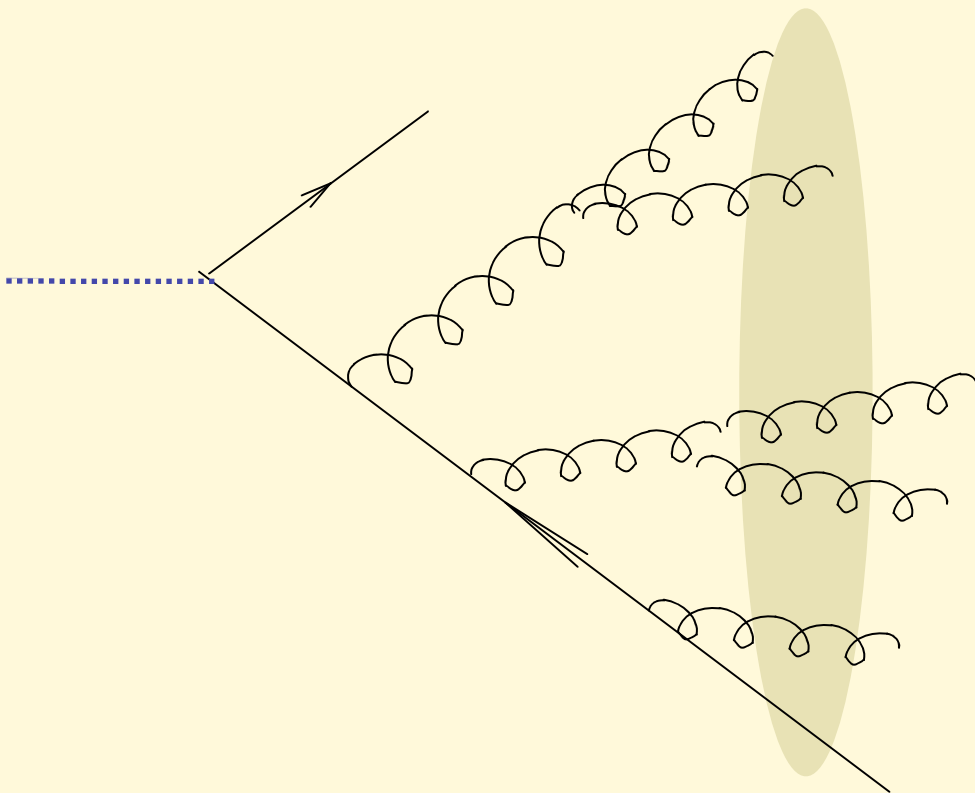
$$\int \frac{d\phi}{2\pi} W_{(j)} = \frac{1}{1 - \cos \theta_{jk}} \text{ if } \theta_{jk} < \theta_{ij}, \quad 0 \text{ otherwise}$$

Further branchings will obey angular ordering relative to the new angles. As a result emission angles get smaller and smaller, squeezing the jet

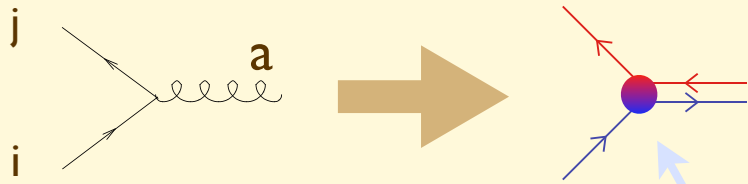




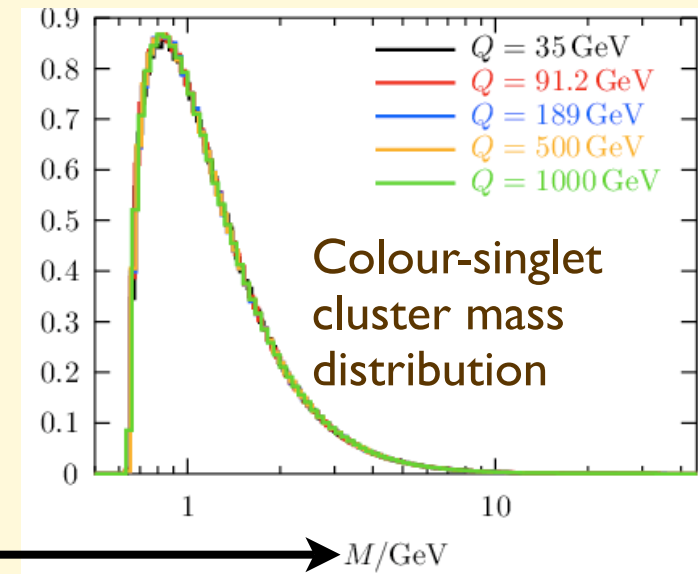
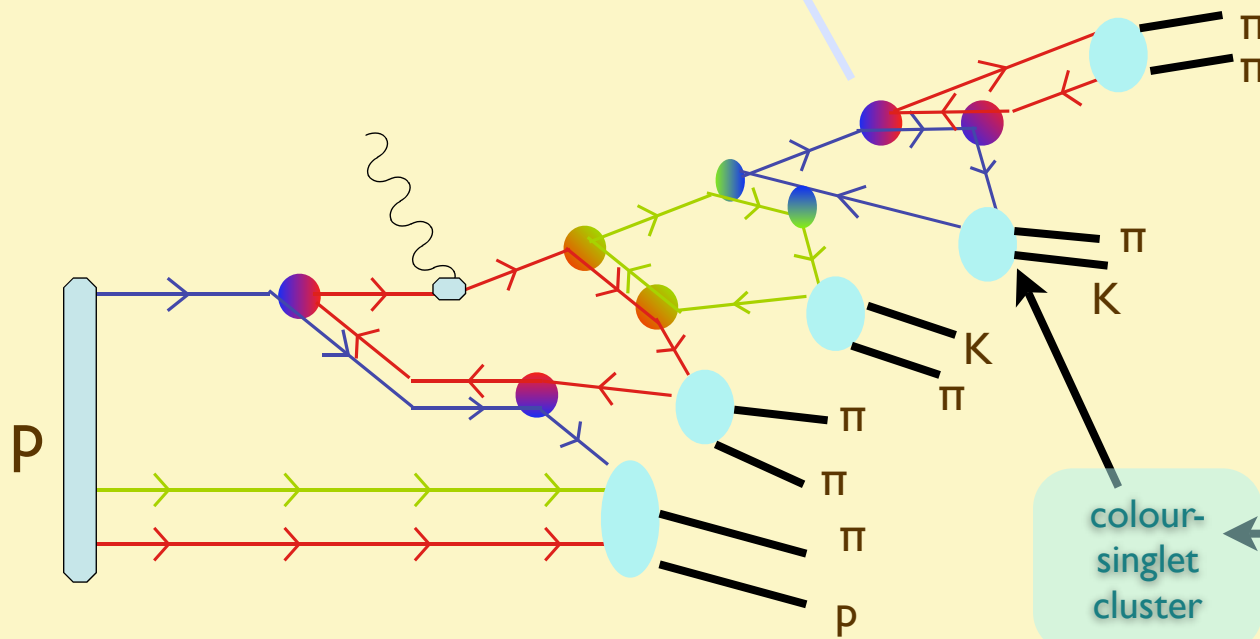
The construction can be iterated to the next emission, with the result that emission angles keep getting smaller and smaller => **jet structure**

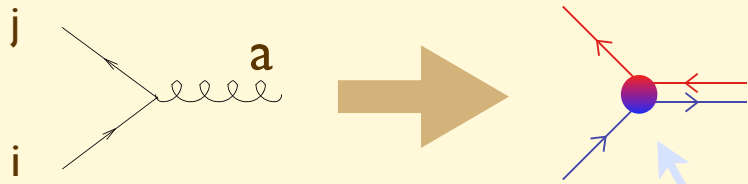


Total colour charge of the system is equal to the quark colour charge. Treating the system as the incoherent superposition of  $N$  gluons would lead to artificial growth of gluon multiplicity. Angular ordering enforces coherence, and leads to the proper evolution with energy of particle multiplicities.

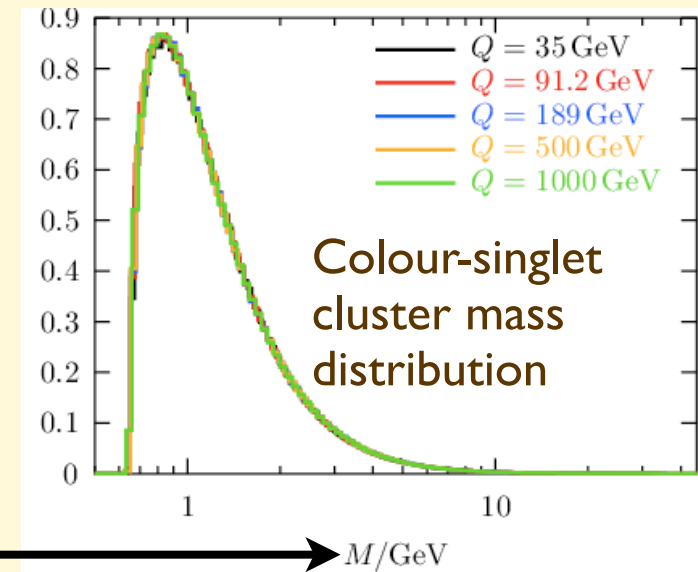
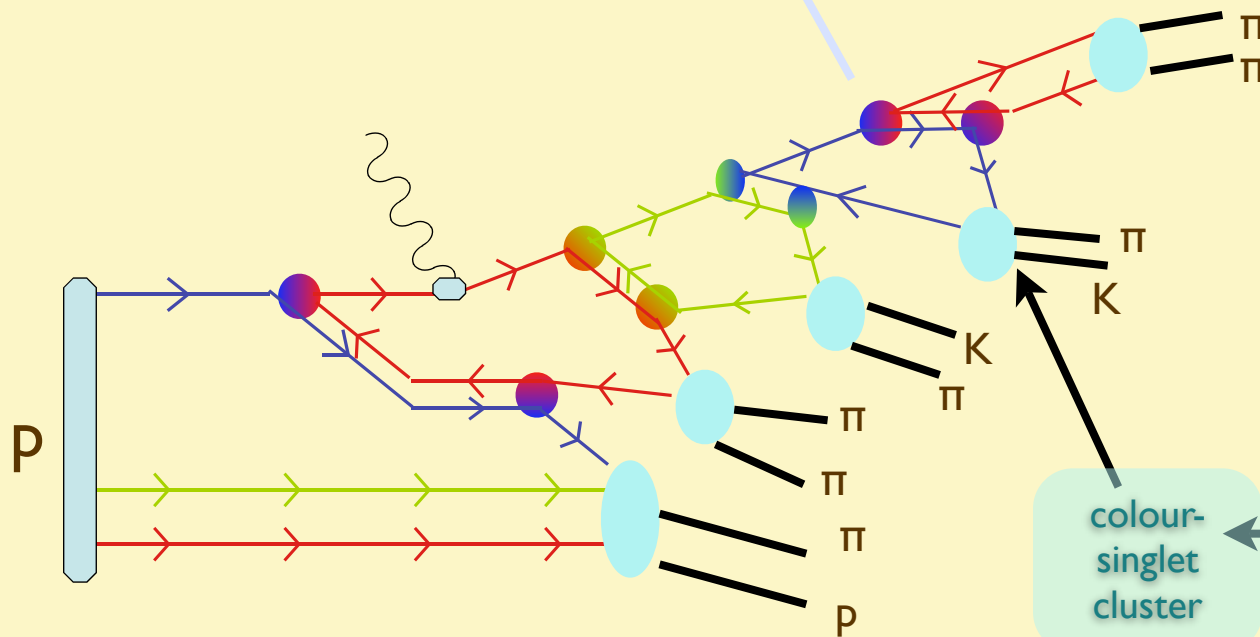


The structure of the perturbative evolution leads naturally to the clustering in phase-space of colour-singlet parton pairs ("preconfinement"). Long-range correlations are strongly suppressed. Hadronization will only act locally, on low-mass colour-singlet clusters.

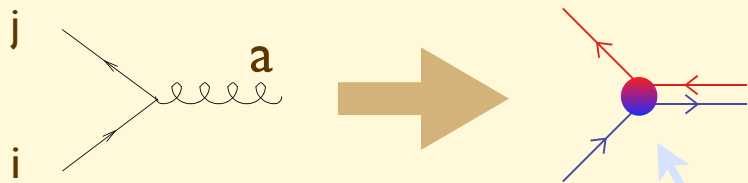




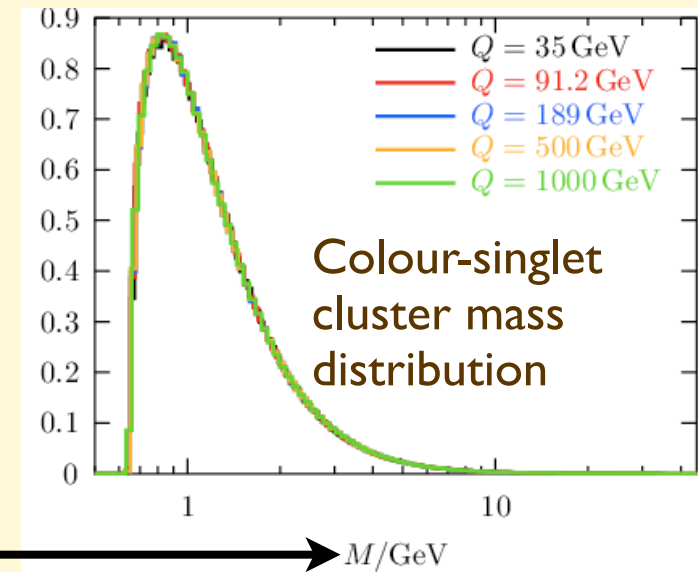
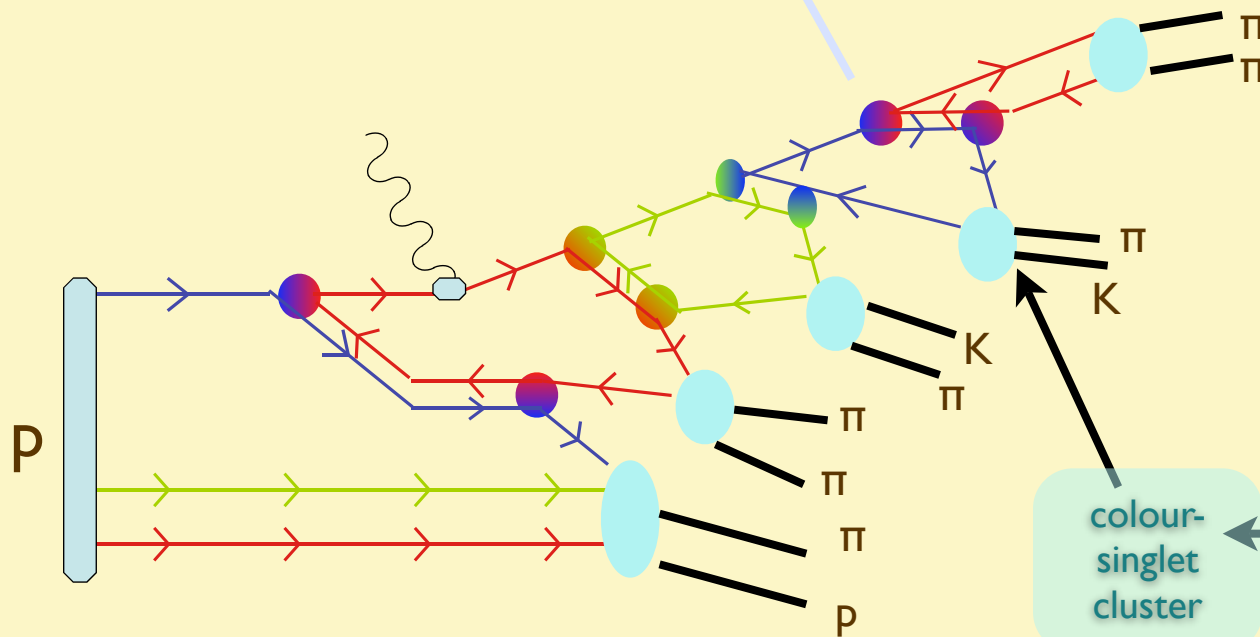
The structure of the perturbative evolution leads naturally to the clustering in phase-space of colour-singlet parton pairs ("preconfinement"). Long-range correlations are strongly suppressed. Hadronization will only act locally, on low-mass colour-singlet clusters.



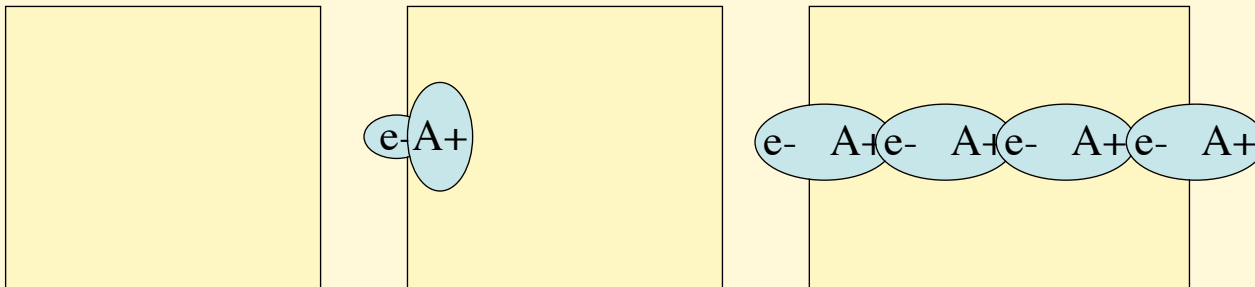
Colour is left "behind" by the struck quark. The first soft gluon emitted at large angle will connect to the beam fragments, ensuring that the beam fragments can recombine to form hadrons, and will allow the struck quark to evolve without having to worry about what happens to the proton fragments.

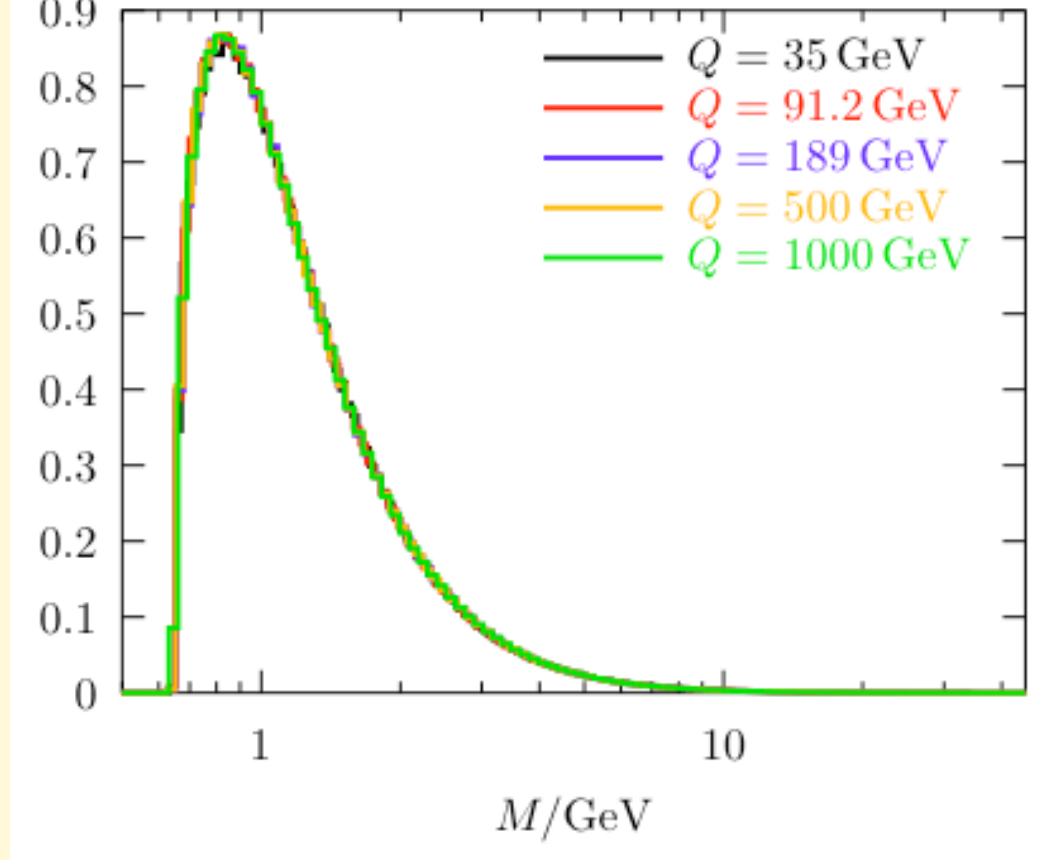


The structure of the perturbative evolution leads naturally to the clustering in phase-space of colour-singlet parton pairs ("preconfinement"). Long-range correlations are strongly suppressed. Hadronization will only act locally, on low-mass colour-singlet clusters.



Colour is left "behind" by the struck quark. The first soft gluon emitted at large angle will connect to the beam fragments, ensuring that the beam fragments can recombine to form hadrons, and will allow the struck quark to evolve without having to worry about what happens to the proton fragments.



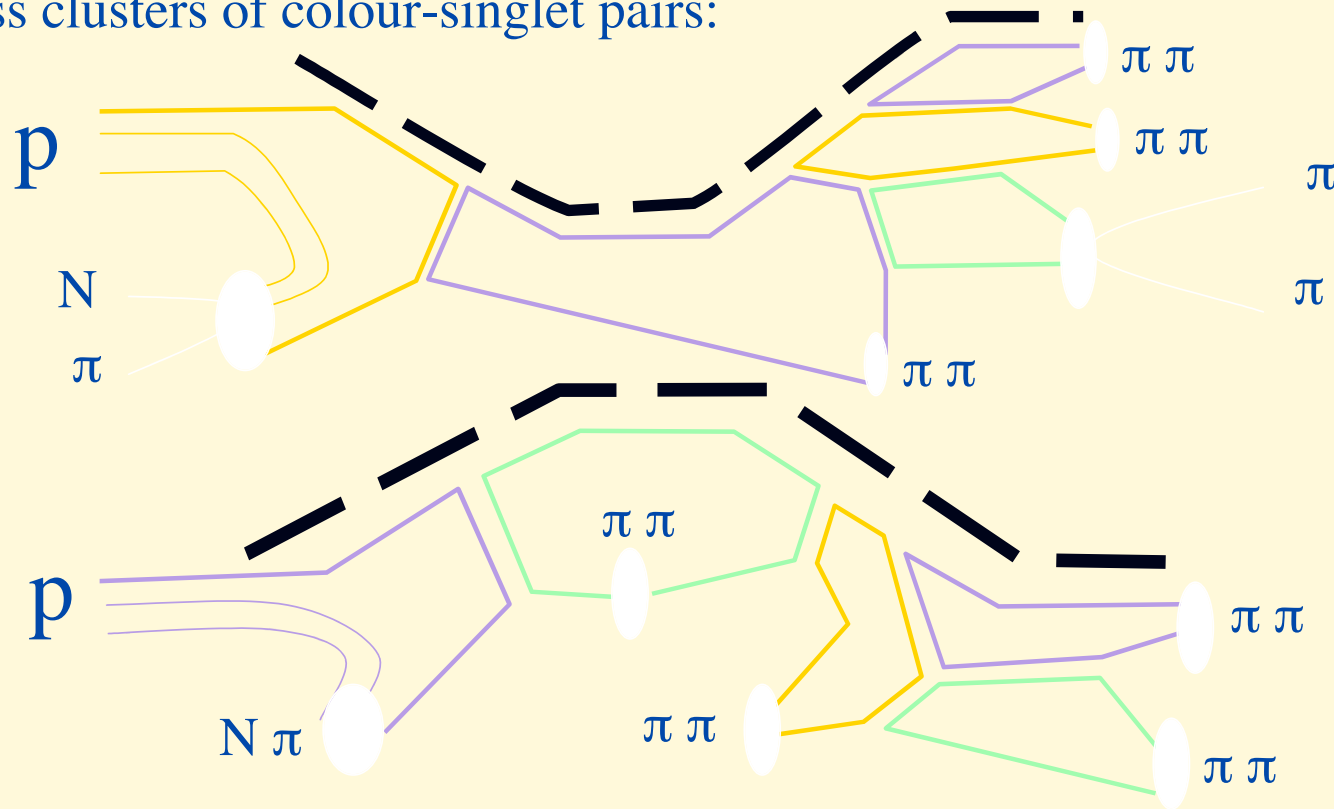


The existence of high-mass clusters, however rare, is unavoidable, due to IR cutoff which leads to a non-zero probability that no emission takes place. This is particularly true for evolution of massive quarks (as in, e.g.  $Z \rightarrow b\bar{b}$  or  $c\bar{c}$ ). Prescriptions have to be defined to deal with the “evolution” of these clusters. **This has an impact on the  $z \rightarrow \mathbf{r}$  behaviour of fragmentation functions.**

Phenomenologically, this leads to uncertainties, for example, in the background rates for  $H \rightarrow \gamma\gamma$  ( $\text{jet} \rightarrow \gamma$ ).

# Hadronization

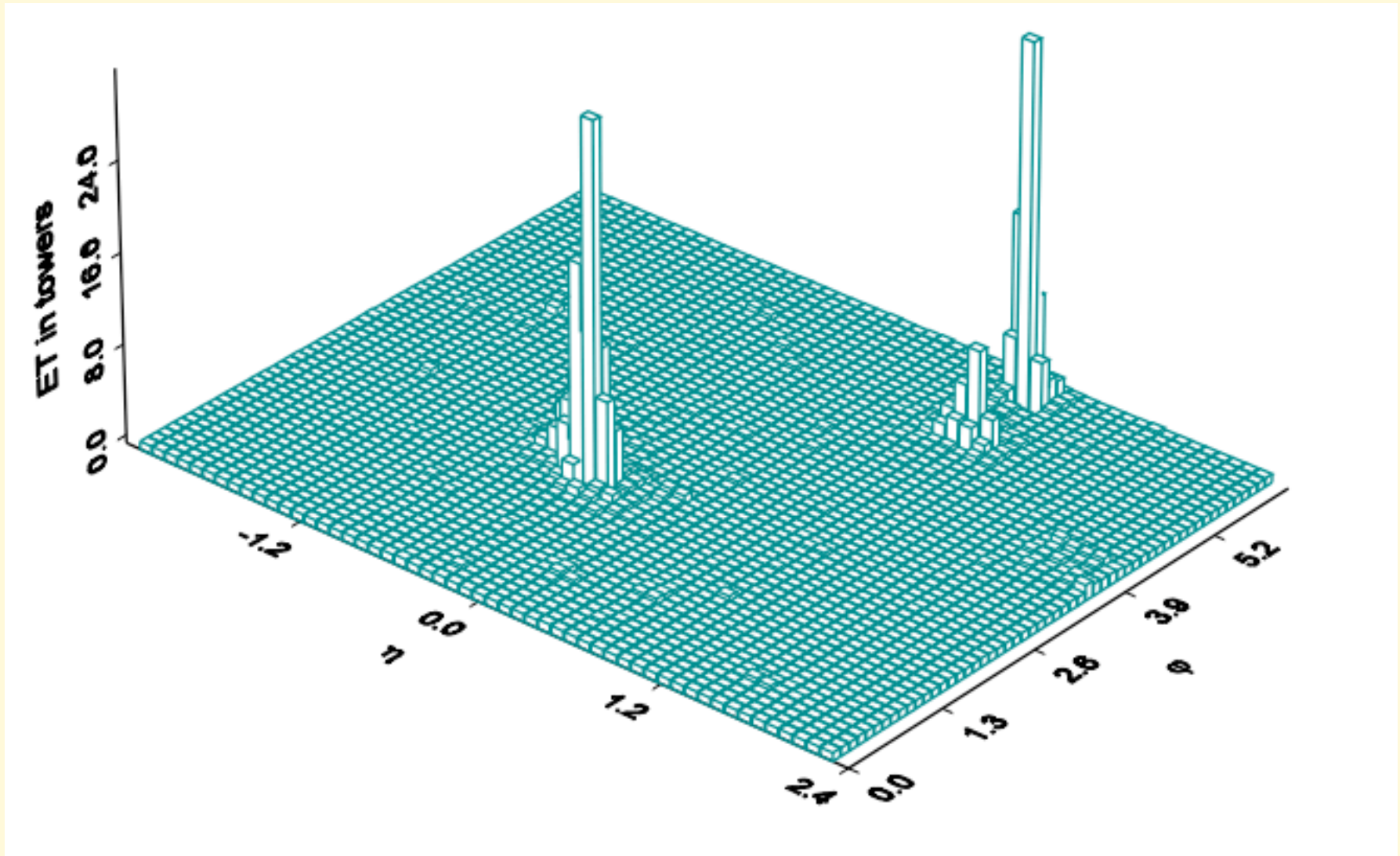
At the end of the perturbative evolution, the final state consists of quarks and gluons, forming, as a result of angular-ordering, low-mass clusters of colour-singlet pairs:



Thanks to the cluster pre-confinement, hadronization is local and independent of the nature of the primary hard process, as well as of the details of how hadronization acts on different clusters. Among other things, one therefore expects:

$$\mathbf{N(pions) = C \, N(gluons),}$$
$$\mathbf{C=constant \sim 2}$$

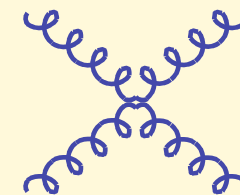
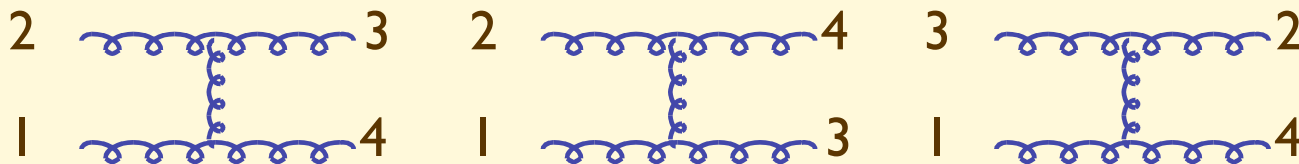
# Jets in hadronic collisions



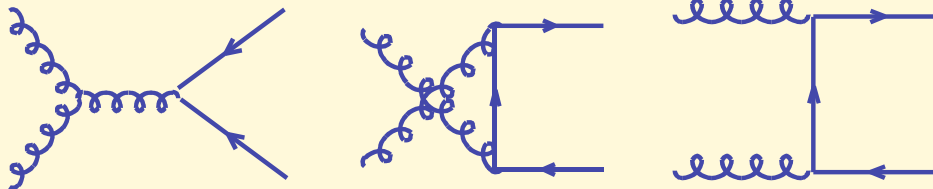
- Inclusive production of jets is the largest component of high- $Q$  phenomena in hadronic collisions
- QCD predictions are known up to NLO accuracy
- Intrinsic theoretical uncertainty (at NLO) is approximately 10%
- Uncertainty due to knowledge of parton densities varies from 5-10% (at low transverse momentum,  $p_T$  to 100% (at very high  $p_T$  corresponding to high- $x$  gluons)
- Jet are used as probes of the quark structure (possible substructure implies departures from point-like behaviour of cross-section), or as probes of new particles (peaks in the invariant mass distribution of jet pairs)



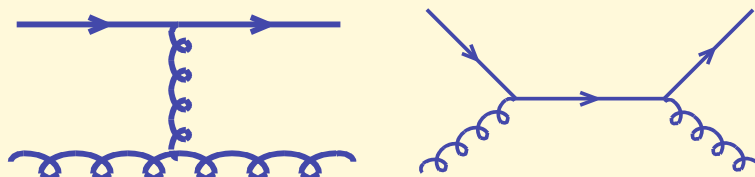
$gg \rightarrow gg$



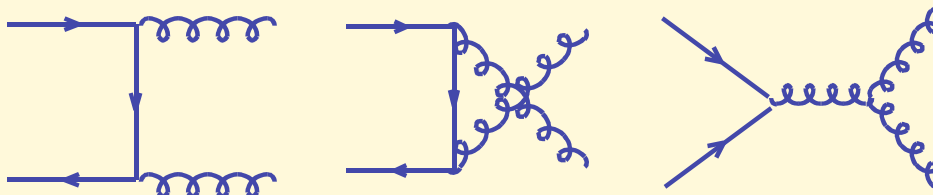
$gg \rightarrow q\bar{q}$



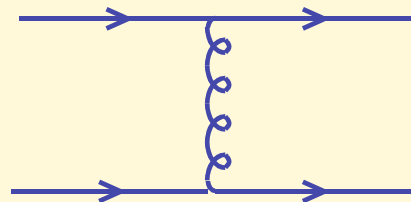
$qg \rightarrow qg$



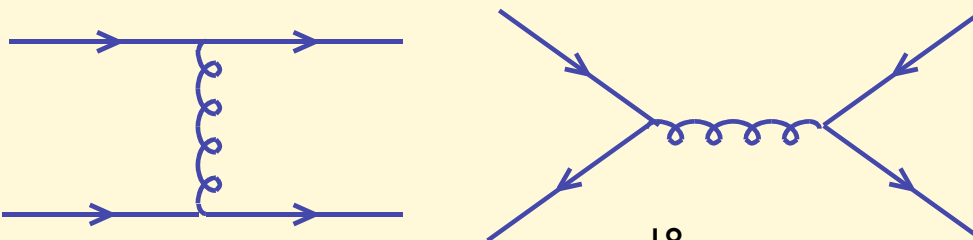
$q\bar{q} \rightarrow gg$



$qq' \rightarrow qq'$



$q\bar{q} \rightarrow q\bar{q}$



# Phase space and cross-section for LO jet production

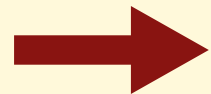
$$d[PS] = \frac{d^3 p_1}{(2\pi)^2 2p_1^0} \frac{d^3 p_2}{(2\pi)^2 2p_2^0} (2\pi)^4 \delta^4(P_{in} - P_{out}) dx_1 dx_2$$

$$(a) \quad \delta(E_{in} - E_{out}) \delta(P_{in}^z - P_{out}^z) dx_1 dx_2 = \frac{1}{2E_{beam}^2}$$

$$(b) \quad \frac{dp^z}{p^0} = dy \equiv d\eta$$



$$d[PS] = \frac{1}{4\pi S} p_T dp_T d\eta_1 d\eta_2$$



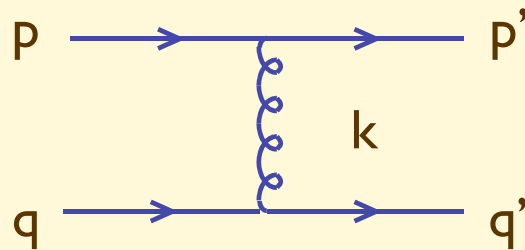
$$\frac{d^3 \sigma}{dp_T d\eta_1 d\eta_2} = \frac{p_T}{4\pi S} \sum_{i,j} f_i(x_1) f_j(x_2) \frac{1}{2\hat{s}} \sum_{kl} \overline{|M(ij \rightarrow kl)|^2}$$

The measurement of  $p_T$  and rapidities for a dijet final state uniquely determines the parton momenta  $x_1$  and  $x_2$ . Knowledge of the partonic cross-section allows therefore the determination of partonic densities  $f(x)$

## Small-angle jet production, a useful approximation for the determination of the matrix elements and of the cross-section

At small scattering angle,  $t = (p_1 - p_3)^2 \sim (1 - \cos \theta) \rightarrow 0$

and the  $1/t^2$  propagators associated with t-channel gluon exchange dominate the matrix elements for all processes. In this limit it is easy to evaluate the matrix elements. For example:



$$\sim (\lambda^a)_{ij} (\lambda^a)_{kl} (2p_\mu) \frac{1}{t} (2q_\mu) = \frac{2s}{t} (\lambda^a)_{ij} (\lambda^a)_{kl}$$

where we used the fact that, for  $k=p-p' \ll p$  (small angle scattering),

$$\bar{u}(p') \gamma_\mu u(p) \sim \bar{u}(p) \gamma_\mu u(p) = 2p_\mu$$

Using our colour algebra results, we then get:

$$\overline{\sum_{col, spin}} |M|^2 = \frac{1}{N_c^2} \frac{N_c^2 - 1}{4} \frac{4s^2}{t^2}$$

Noting that the result must be symmetric under  $s \leftrightarrow u$  exchange, and setting

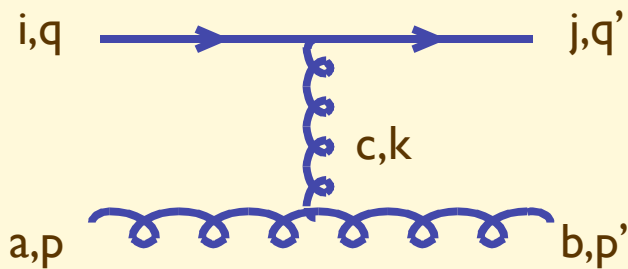
$N_c=3$ , we finally obtain:

$$\overline{\sum_{col, spin}} |M|^2 = \frac{4}{9} \frac{s^2 + u^2}{t^2}$$

which turns out to be the exact result!

# Quark-gluon and gluon-gluon scattering

We repeat the exercise in the more complex case of qg scattering, assuming the dominance of the t-channel gluon-exchange diagram:



$$\sim f^{abc} \lambda_{ij}^c 2p_\mu \frac{1}{t} 2q_\mu = 2 \frac{s}{t} f^{abc} \lambda_{ij}^c$$

Using the colour algebra results, and enforcing the  $s \leftrightarrow u$  symmetry, we get:

$$\overline{\sum_{col, spin}} |M|^2 = \frac{s^2 + u^2}{t^2}$$

which differs by only 20% from the exact result even in the large-angle region, at  $90^\circ$

$$\overline{\sum_{col, spin}} |M|^2 = \frac{s^2 + u^2}{t^2} - \frac{4}{9} \frac{s^2 + u^2}{us}$$

In a similar way we obtain for gg scattering (using the  $t \leftrightarrow u$  symmetry):

$$\overline{\sum_{col, spin}} |M(gg \rightarrow gg)|^2 = \frac{9}{2} \left( \frac{s^2}{t^2} + \frac{s^2}{u^2} \right)$$

compared to the exact result

$$\overline{\sum_{col, spin}} |M(gg \rightarrow gg)|^2 = \frac{9}{2} \left( 3 - \frac{ut}{s^2} - \frac{us}{t^2} - \frac{st}{u^2} \right)$$

with a 20% difference at  $90^\circ$

Note that in the leading  $1/t$  approximation we get the following result:

$$\hat{\sigma}_{gg} : \hat{\sigma}_{qg} : \hat{\sigma}_{qq} = \frac{9}{4} : 1 : \frac{4}{9}$$

where  $4/9 = C_F / C_A = [(N^2-1)/2N] / N$  is the ratio of the squared colour charges of quarks and gluons

and therefore

$$d\sigma_{jet} = \int dx_1 dx_2 \sum_{ij} f_i(x_1) f_j(x_2) d\hat{\sigma}_{ij} = \int dx_1 dx_2 \sum_{ij} F(x_1) F(x_2) d\hat{\sigma}_{gg}$$

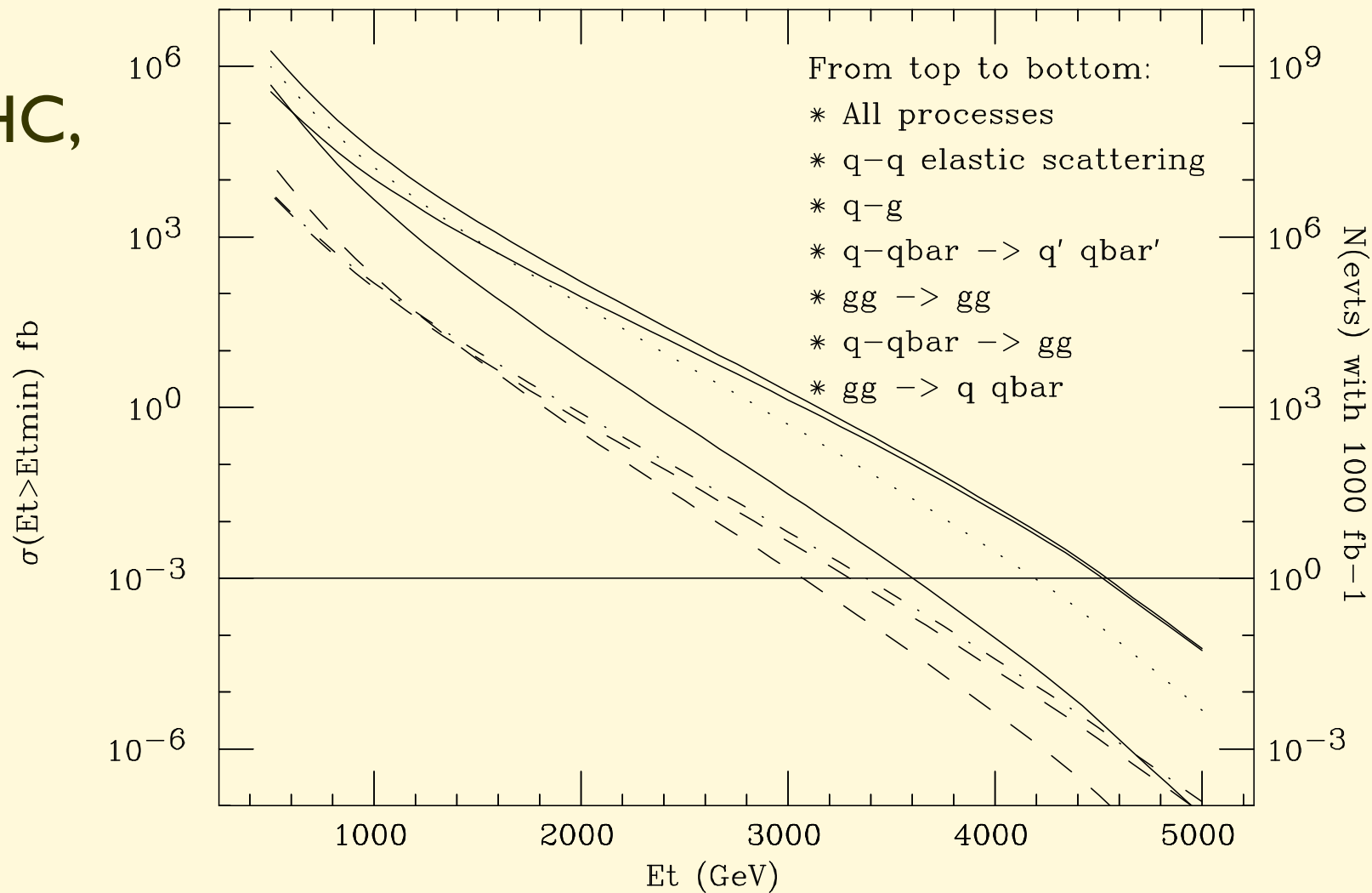
where we defined the 'effective parton density'  $F(x)$ :

$$F(x) = g(x) + \frac{4}{9} \sum_i [q_i(x) + \bar{q}_i(x)]$$

As a result jet data cannot be used to extract separately gluon and quark densities. On the other hand, assuming an accurate knowledge of the quark densities (say from HERA), jet data can help in the determination of the gluon density

Process	$\frac{d\hat{\sigma}}{d\Phi_2}$	at 90°
$qq' \rightarrow qq'$	$\frac{4}{9} \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}$	2.22
$qq \rightarrow qq$	$\left[ \frac{4}{9} \left( \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} + \frac{\hat{s}^2 + \hat{t}^2}{\hat{u}^2} \right) - \frac{8}{27} \frac{\hat{s}^2}{\hat{u}\hat{t}} \right]$	3.26
$q\bar{q} \rightarrow q'\bar{q}'$	$\frac{4}{9} \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2}$	0.22
$q\bar{q} \rightarrow q\bar{q}$	$\left[ \frac{4}{9} \left( \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} + \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2} \right) - \frac{8}{27} \frac{\hat{u}^2}{\hat{s}\hat{t}} \right]$	2.59
$q\bar{q} \rightarrow gg$	$\left[ \frac{32}{27} \frac{\hat{t}^2 + \hat{u}^2}{\hat{t}\hat{u}} - \frac{8}{3} \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2} \right]$	1.04
$gg \rightarrow q\bar{q}$	$\left[ \frac{1}{6} \frac{\hat{t}^2 + \hat{u}^2}{\hat{t}\hat{u}} - \frac{3}{8} \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2} \right]$	0.15
$gq \rightarrow gq$	$\left[ -\frac{4}{9} \frac{\hat{s}^2 + \hat{u}^2}{\hat{s}\hat{u}} + \frac{\hat{u}^2 + \hat{s}^2}{\hat{t}^2} \right]$	6.11
$gg \rightarrow gg$	$\frac{9}{2} \left( 3 - \frac{\hat{t}\hat{u}}{\hat{s}^2} - \frac{\hat{s}\hat{u}}{\hat{t}^2} - \frac{\hat{s}\hat{t}}{\hat{u}^2} \right)$	30.4

# Jet production rates at the LHC, subprocess composition



The presence of a quark substructure would manifest itself via contact interactions (as in Fermi's theory of weak interactions). On one side these new interactions would lead to an increase in cross-section, on the other they would affect the jets' angular distributions. In the dijet CMF, **QCD implies Rutherford law**, and extra point-like interactions can then be isolated using a fit. With the anticipated statistics of 300 fb-1, **limits on the scale of the new interactions in excess of 40 TeV should be reached** (to increase to 60 TeV with 3000 fb-1)

# Some more kinematics

Prove as an **exercise** that

$$x_{1,2} = \frac{p_T}{E_{beam}} \cosh y^* e^{\pm y_b}$$

where

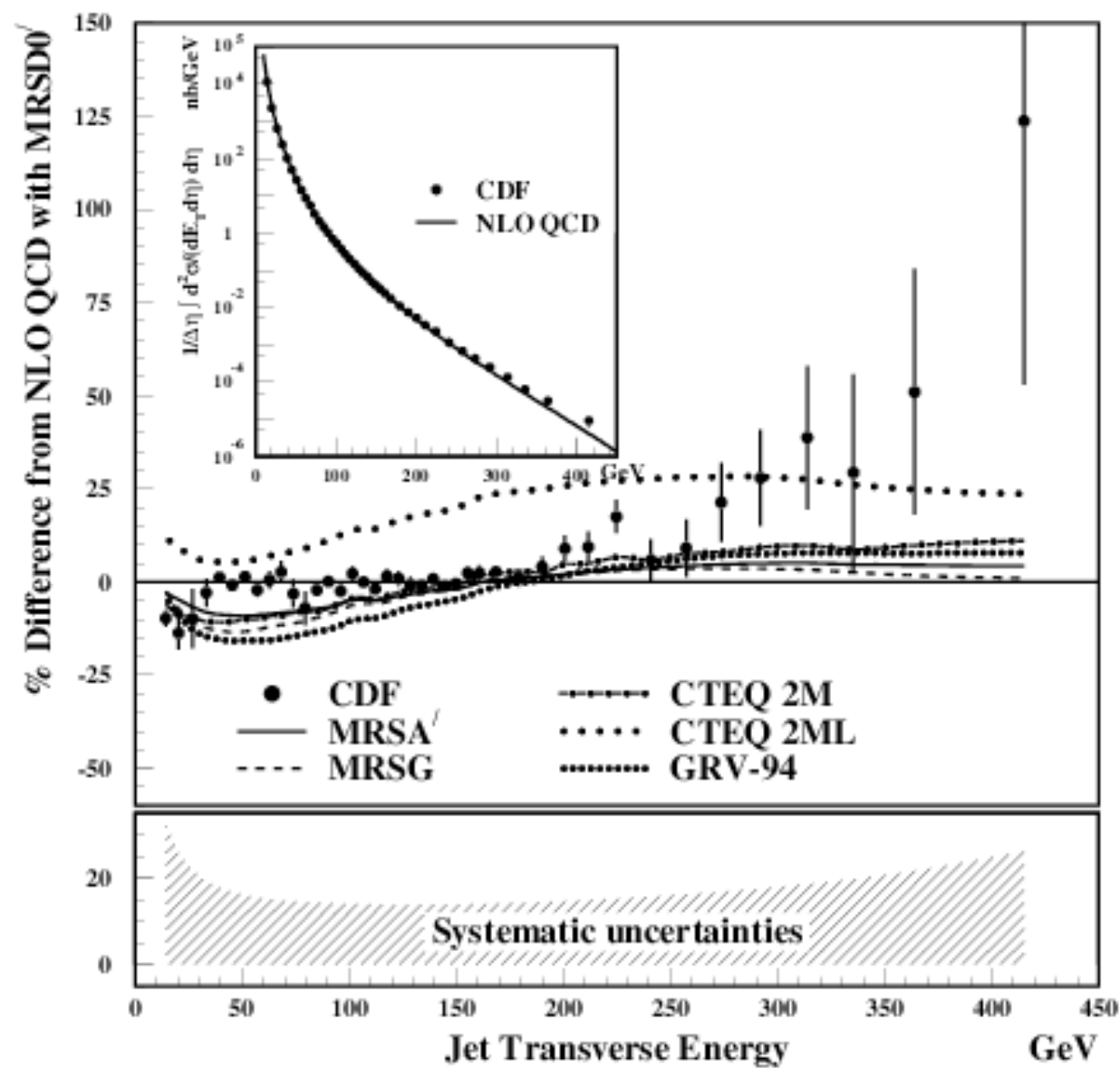
$$y^* = \frac{\eta_1 - \eta_2}{2}, \quad y_b = \frac{\eta_1 + \eta_2}{2}$$

We can therefore reach large values of  $x$  either by selecting large invariant mass events:

$$\frac{p_T}{E_{beam}} \cosh y^* \equiv \sqrt{\tau} \rightarrow 1$$

or by selecting low-mass events, but with large boosts ( $y_b$  large) in either positive or negative directions. In this case, we probe large- $x$  with events where possible new physics is absent, thus setting consistent constraints on the behaviour of the cross-section in the high-mass region, which could hide new phenomena.





# Example, at the Tevatron

