Introduction to hadronic collisions: theoretical concepts and practical tools for the LHC

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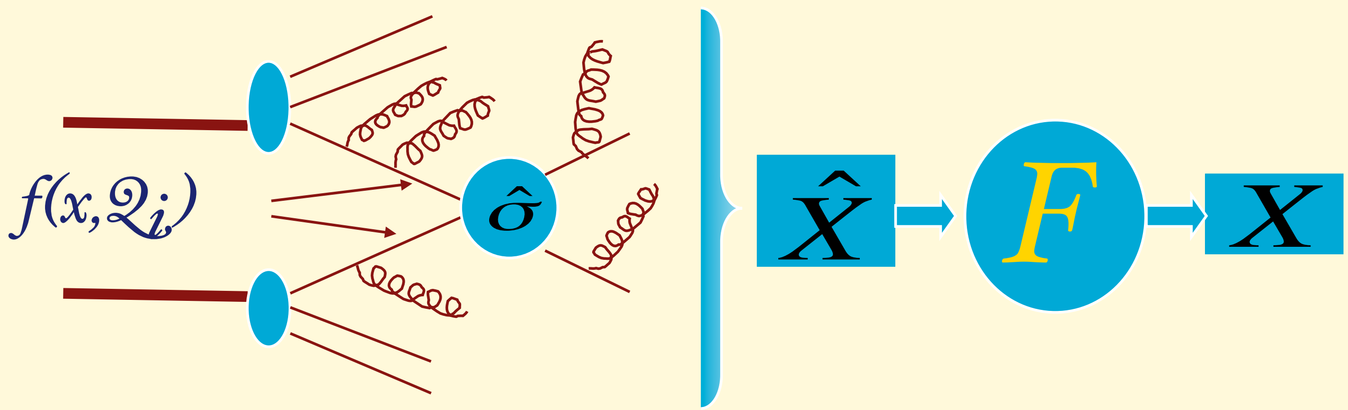
  - Factorization theorem
  - Parton densities
  - Evolution of final states
  - Hard processes
- **Lecture III & IV:** Tools and applications:
  - Monte Carlo codes, virtues and limitations
  - Physics objects relevant to the search of BSM phenomena at the LHC:
    - jets
    - leptons
    - b/c-quark jets
    - W+multijets
    - top quark
  - Example: SUSY searches

Factorization Theorem

$$\frac{d\sigma}{dX} = \sum_{j,k} \int_{\hat{X}} f_j(x_1,Q_i)\, f_k(x_2,Q_i)\, \frac{d\hat{\sigma}_{jk}(Q_i,Q_f)}{d\hat{X}}\, F(\hat{X} \to X; Q_i,Q_f)$$



$$f_j(x,Q)$$

Parton distribution functions (PDF)

- sum over all initial state histories leading, at the scale Q, to:

$$\vec{p}_j = x\,\vec{P}_{proton}$$

$$F(\hat{X} \to X; Q_i,Q_f)$$

- transition from partonic final state to the hadronic observable (hadronization, fragm. function, jet definition, etc)
- Sum over all histories with X in them

Universality of parton densities and factorization, an intuitive view

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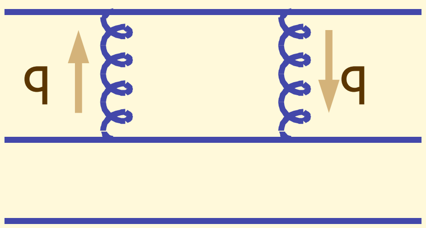
1) Exchange of **hard gluons** among quarks inside the proton is suppressed by powers of $(m_p/Q)^2$



$$\overset{q>Q}{\sim} \int_Q^\infty \frac{d^4q}{q^6} \sim \frac{1}{Q^2}$$

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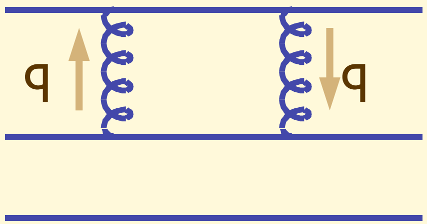

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Assuming asymptotic freedom!

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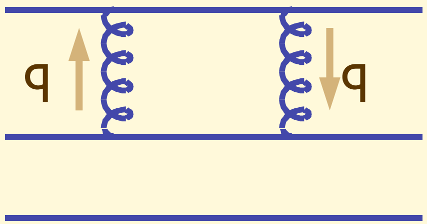
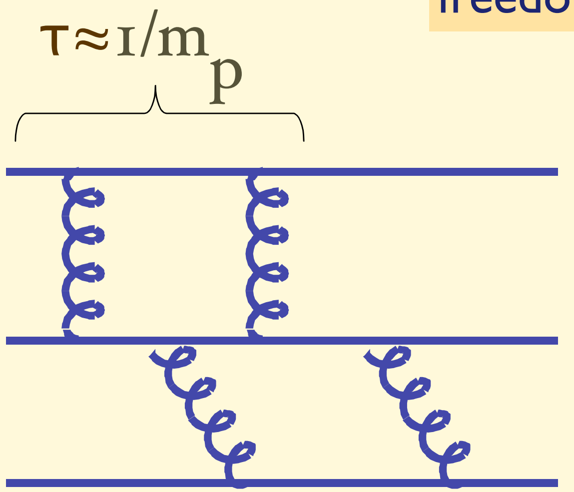


$$\stackrel{q>Q}{\sim} \int_Q^\infty \frac{d^4q}{q^6} \sim \frac{1}{Q^2}$$

Assuming asymptotic freedom!

2) Typical time-scale of interactions binding the proton is therefore of $O(1/m_p)$ (in a frame in which the proton has energy E, $\tau=\gamma/m_p = E/m_p^2$)



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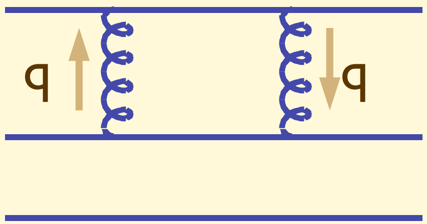

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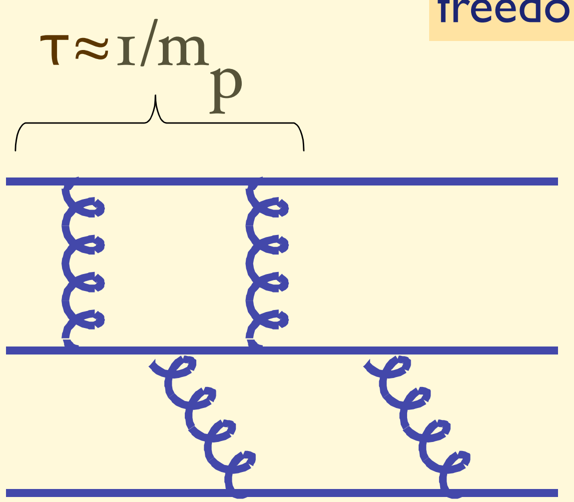
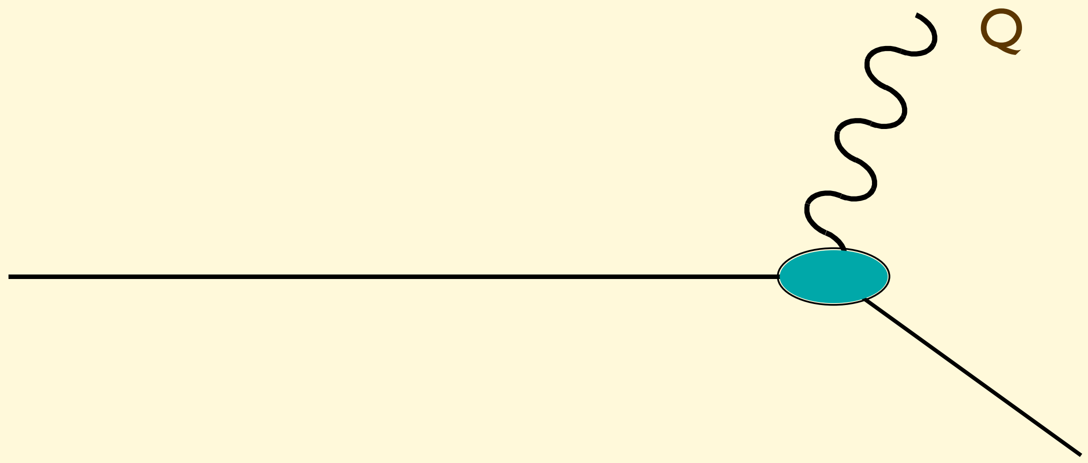


3) If a hard probe ($Q>>m_p$) hits the proton, on a time scale $=1/Q$, there is no time for quarks to negotiate a coherent response. The struck quark receives no feedback from its pals, and acts as a free particle

As a result, to study inclusive processes at large Q it is sufficient to consider the interactions between the external probe and a single parton:



As a result, to study inclusive processes at large Q it is sufficient to consider the interactions between the external probe and a single parton:

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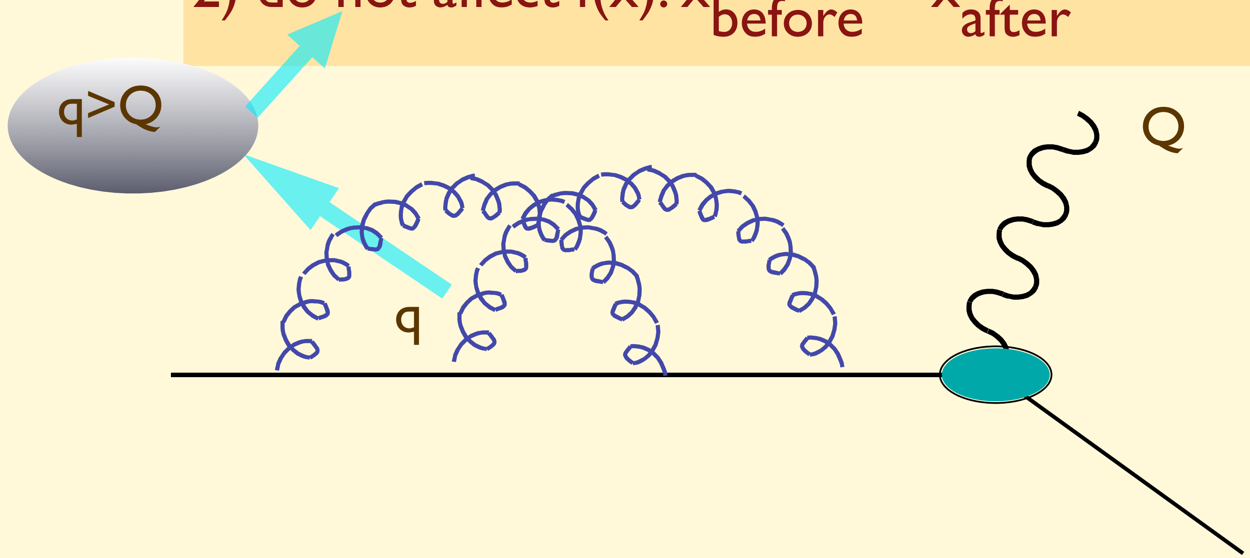
2) do not affect f(x): $x_{before} = x_{after}$



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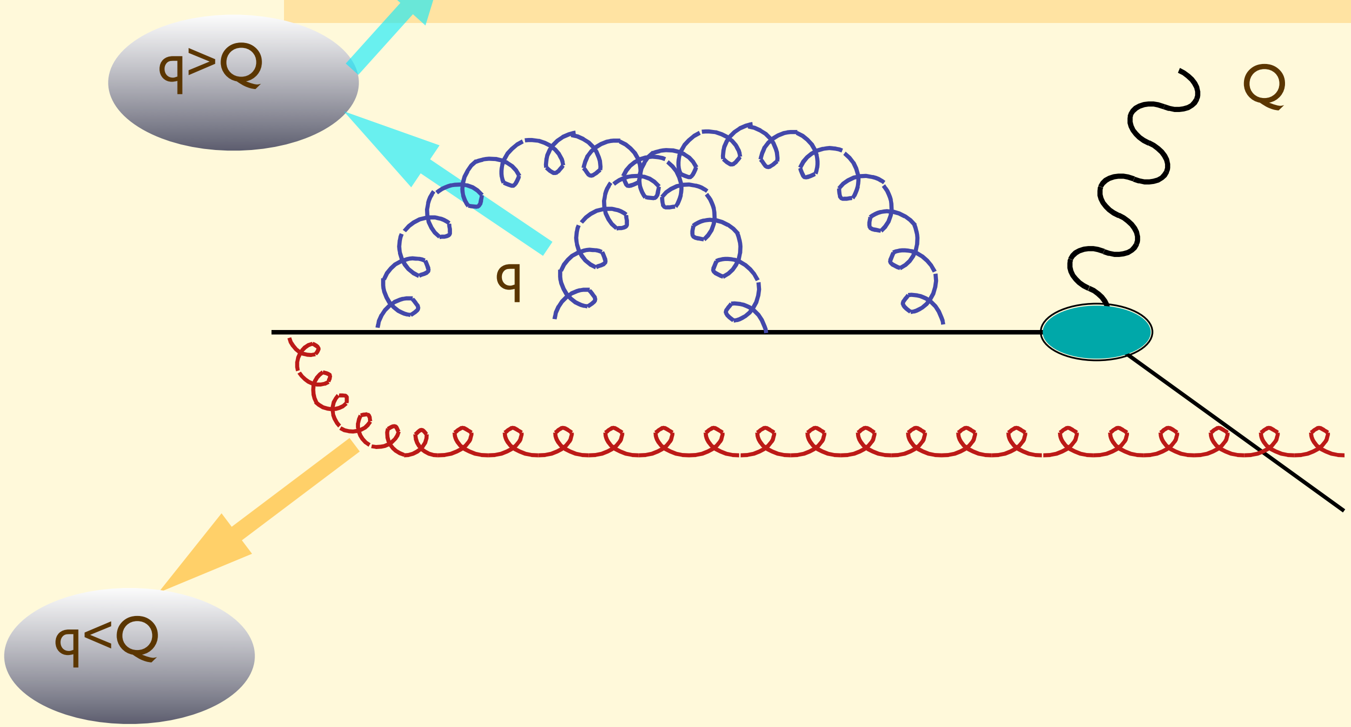

This gluon cannot be reabsorbed because the quark is gone

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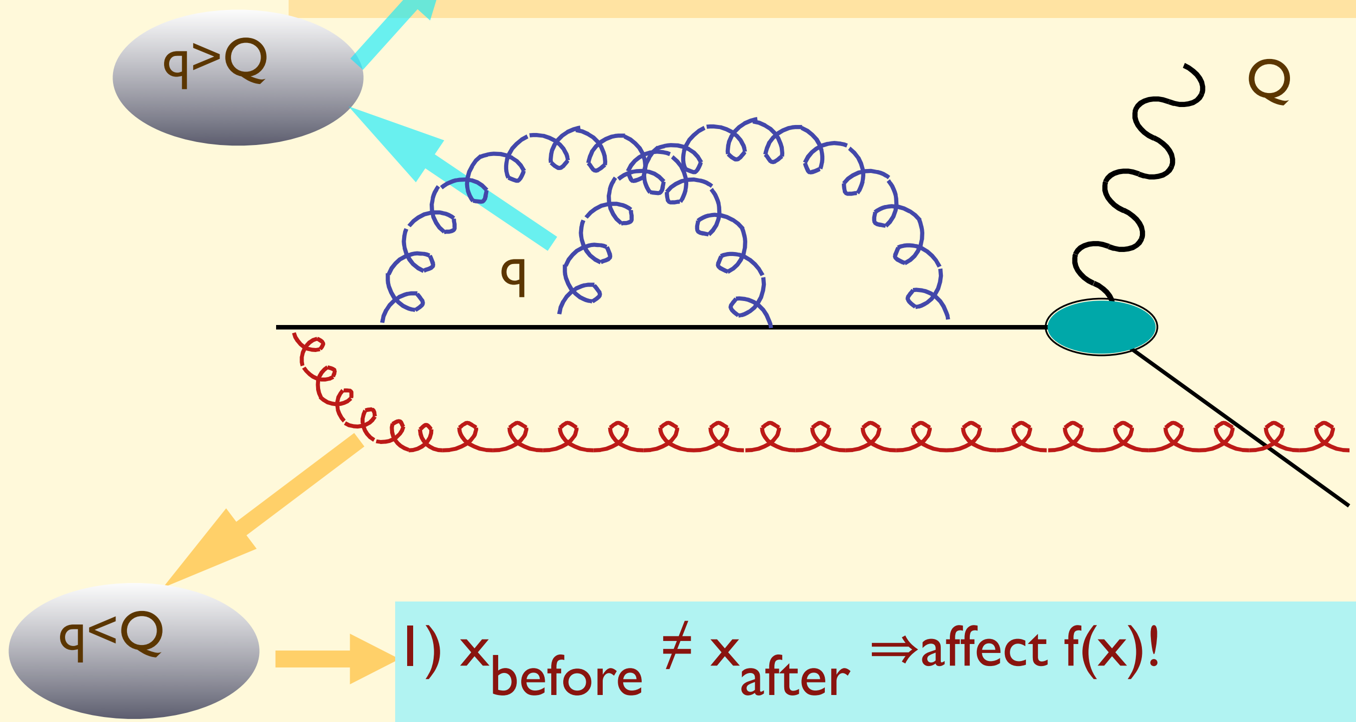

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1) $x_{before} \neq x_{after}$ $\Rightarrow$affect f(x)!

2) for q≈1 GeV **not** calculable in pQCD

As a result, to study inclusive processes at large Q it is sufficient to consider the interactions between the external probe and a single parton:

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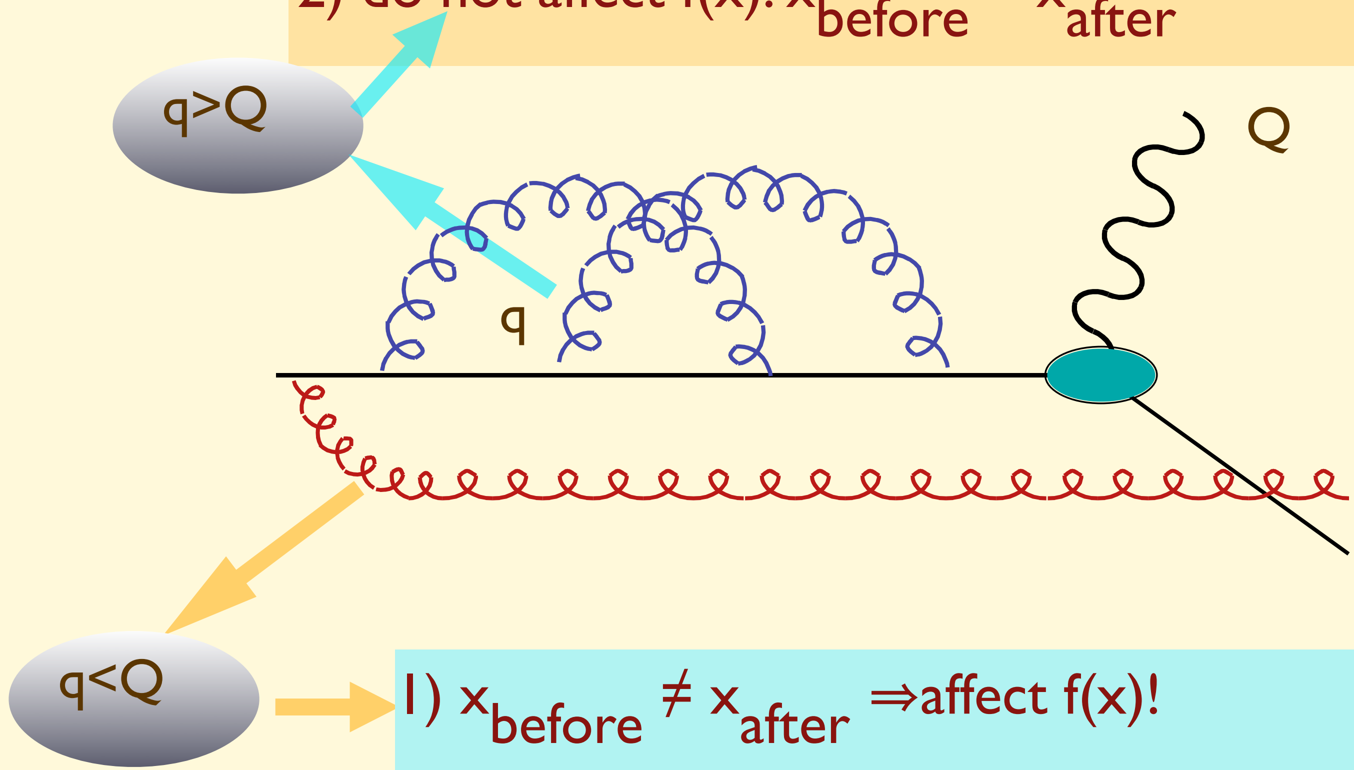

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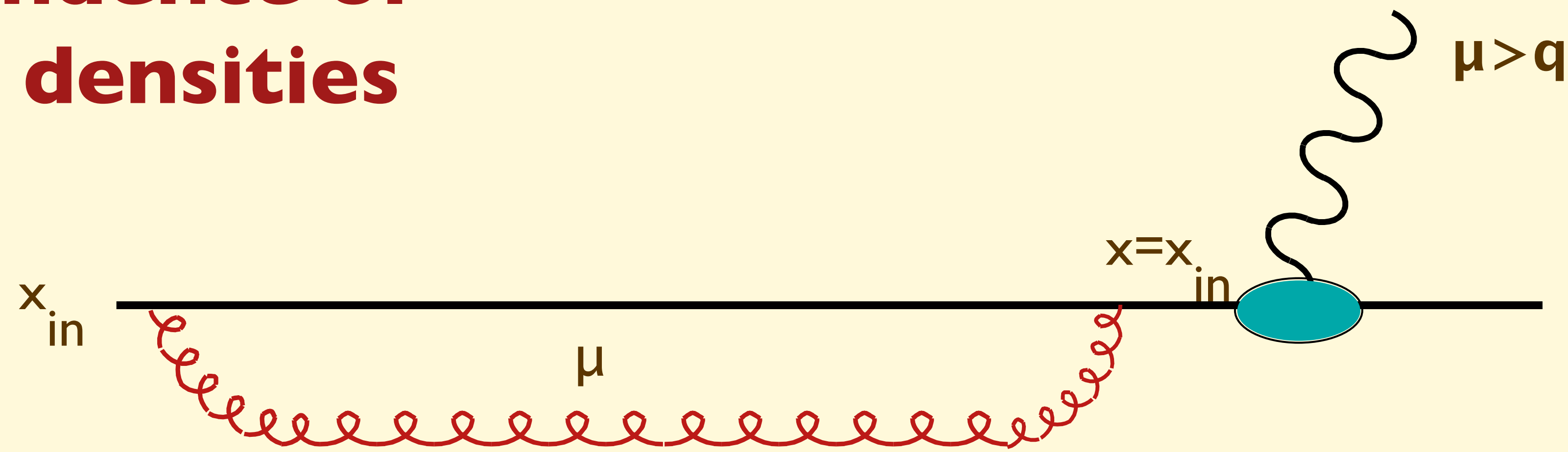
1) $x_{before} \neq x_{after}$ ⇒affect f(x)!

2) for q≈1 GeV **not** calculable in pQCD

However, since τ(q≈1GeV)>>1/Q, the emission of low-virtuality gluons will take place long before the hard collision, and therefore cannot depend on the detailed nature of the hard probe. While it is not calculable in pQCD, f(q<<Q) can be measured using a reference probe, and used elsewhere

➡ **Universality of f(x)**

Q dependence of parton densities



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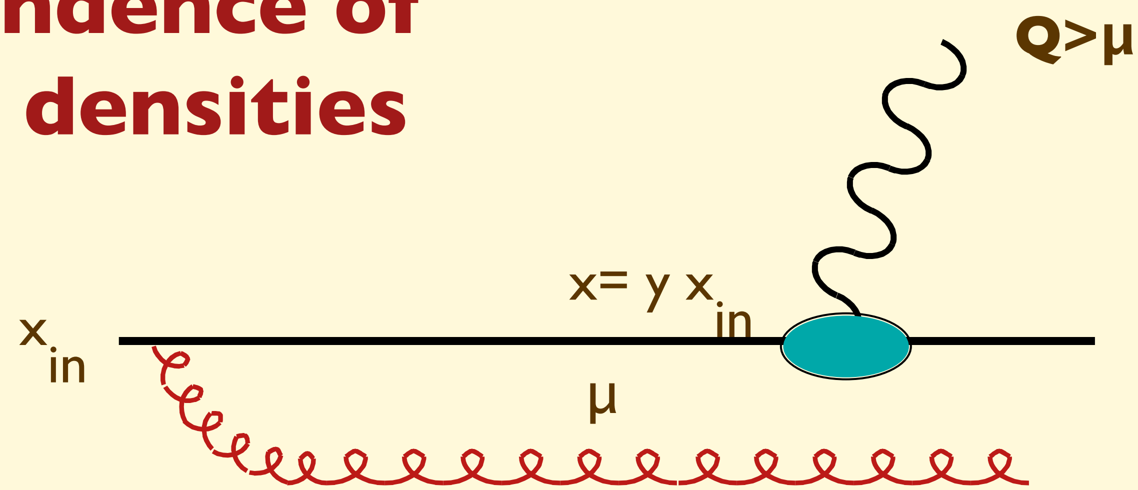

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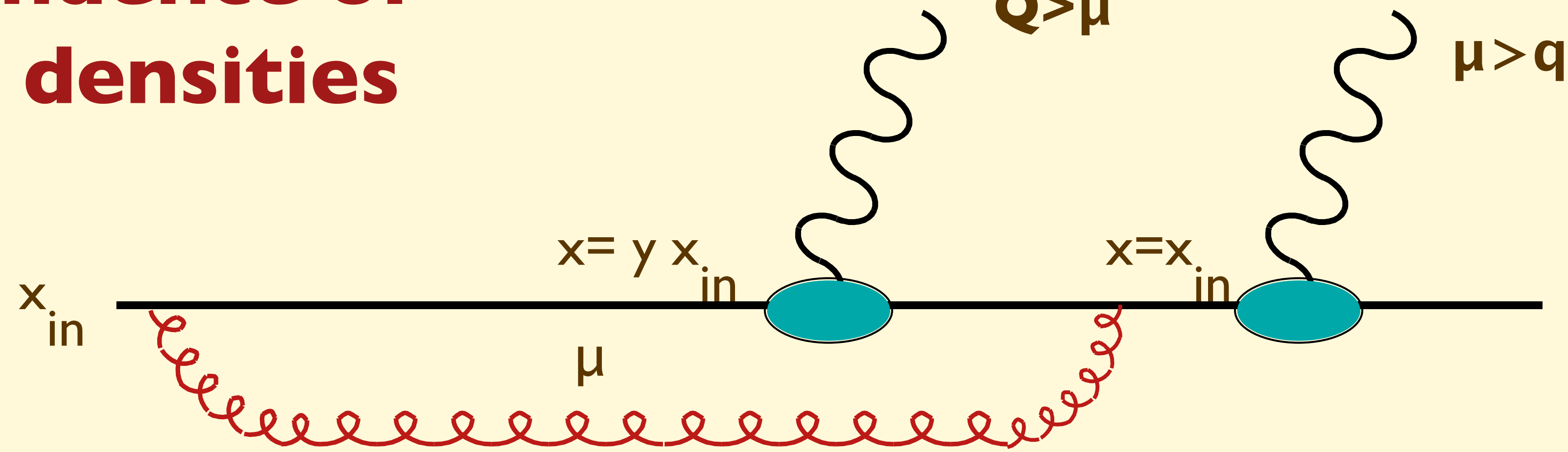

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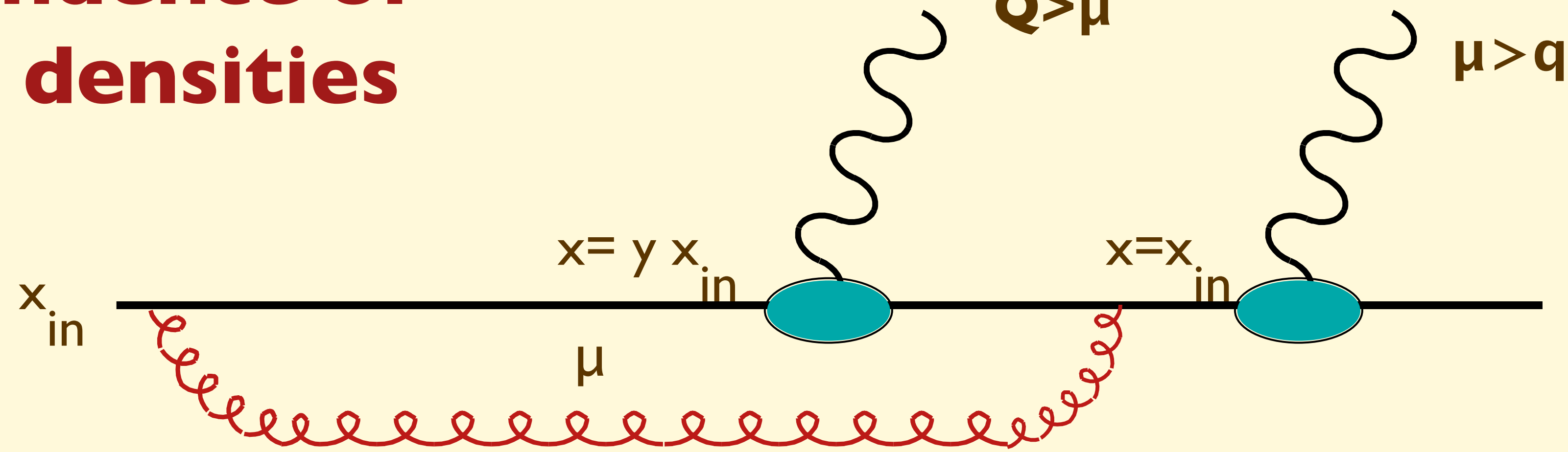


The larger is Q, the more gluons will **not** have time to be reabsorbed

PDF's depend on Q!

$$f(x,Q) = f(x,\mu) + \int_x^1 dx_{in}\, f(x_{in},\mu) \int_\mu^Q dq^2 \int_0^1 dy\, P(y,q^2)\, \delta(x - y x_{in})$$

$$f(x,Q) = f(x,\mu) + \int_x^1 dx_{in} f(x_{in},\mu) \int_\mu^Q dq^2 \int_0^1 dy P(y,q^2)\,\delta(x - yx_{in})$$

f(x,Q) should be independent of the intermediate scale μ considered:

$$\frac{df(x,Q)}{d\mu^2} = 0 \quad \Rightarrow \boxed{\frac{df(x,\mu)}{d\mu^2} = \int_x^1 \frac{dy}{y} f(y,\mu)\, P(x/y,\mu^2)}$$

One can prove that:

calculable in pQCD

$$P(x,Q^2) = \frac{\alpha_s}{2\pi}\frac{1}{Q^2} P(x)$$

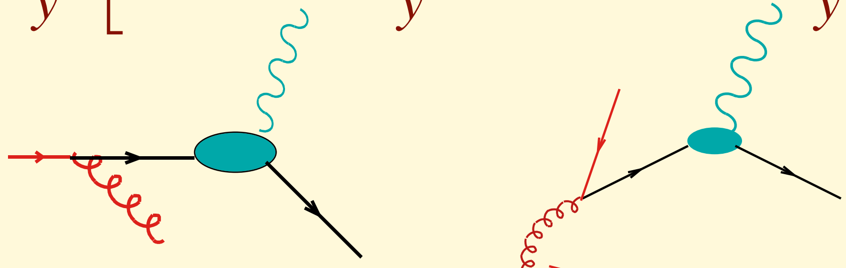
and therefore (Altarelli-Parisi equation):

$$\boxed{\frac{df(x,\mu)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} f(y,\mu)\, P(x/y)}$$

More in general, one should consider additional processes which lead to the evolution of partons at high Q ($t=\log Q^2$):

$$[g(x)]_+ : \quad \int_0^1 dx\, f(x)\, g(x)_+ \equiv \int_0^1 [f(x) - f(1)]\, g(x)\, dx$$

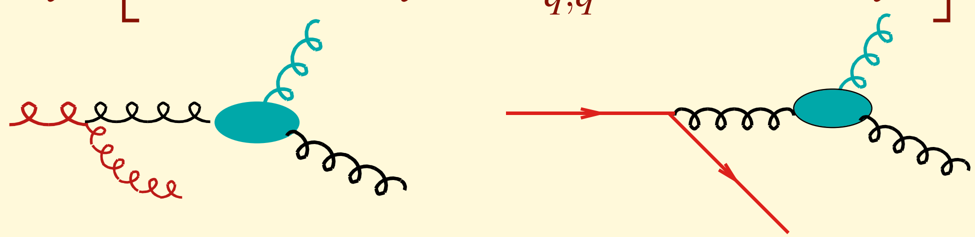
$$\frac{dq(x,Q)}{dt} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[q(y,Q)\, P_{qq}(\frac{x}{y}) + g(y,Q)\, P_{qg}(\frac{x}{y}) \right]$$



$$P_{qq}(x) = C_F \left(\frac{1+x^2}{1-x} \right)_+$$

$$P_{qg}(x) = \frac{1}{2}\left[x^2 + (1-x)^2\right]$$

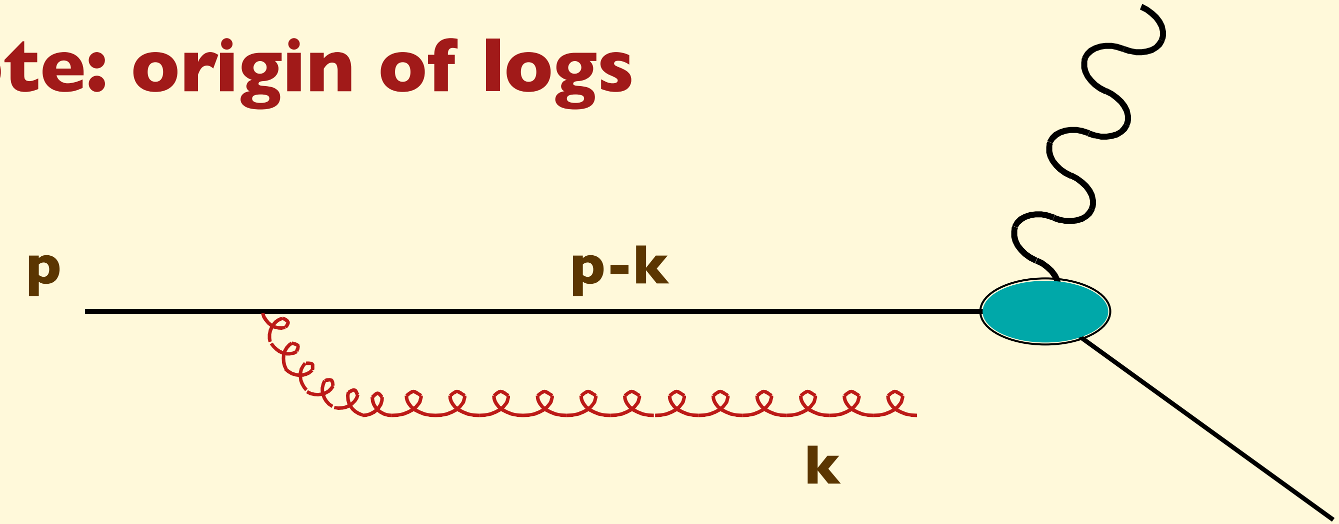
$$\frac{dg(x,Q)}{dt} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[g(y,Q)\, P_{gg}(\frac{x}{y}) + \sum_{q,\bar{q}} q(y,Q)\, P_{gq}(\frac{x}{y}) \right]$$



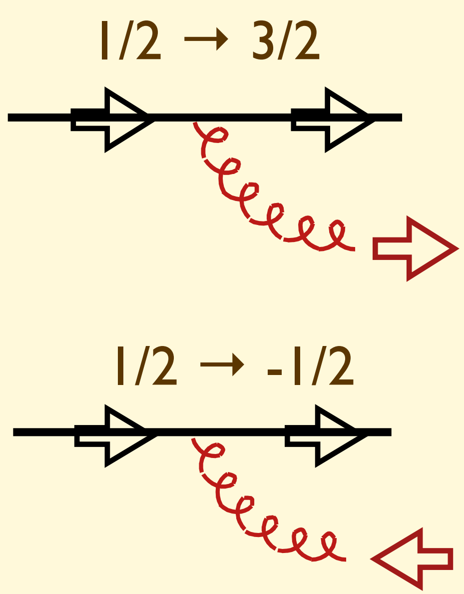
$$P_{gq}(x) = C_F \left(\frac{1+(1-x)^2}{x} \right)$$

$$P_{gg}(x) = 2N_c \left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right] + \delta(1-x) \left(\frac{11N_c - 2n_f}{6} \right)$$

Note: origin of logs



$$(p-k)^2 \;=\; -2\,p^0 k^0 (\,1 - \cos\theta_{pk}\,)$$



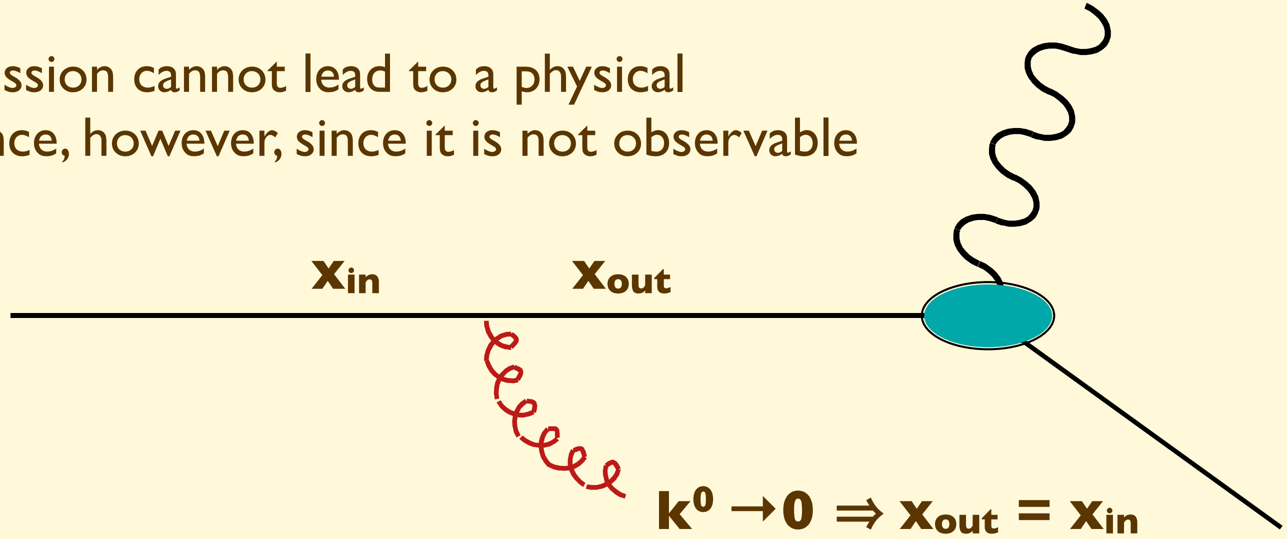
Helicity conservation ~ p · k

$$|M|^2 \sim \left[\frac{1}{(p-k)^2}\right]^2 \times (p \cdot k) \quad \rightarrow \quad \frac{1}{p^0}\frac{dk^0}{k^0}\,\frac{d\theta}{\theta}$$

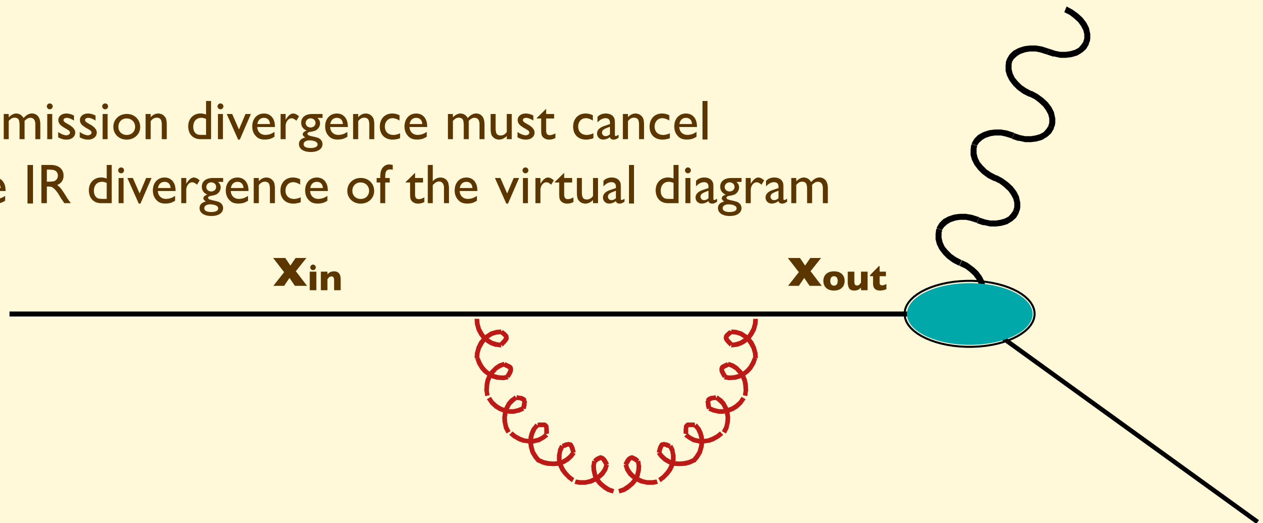
Soft divergence

Collinear divergence

Soft emission cannot lead to a physical divergence, however, since it is not observable

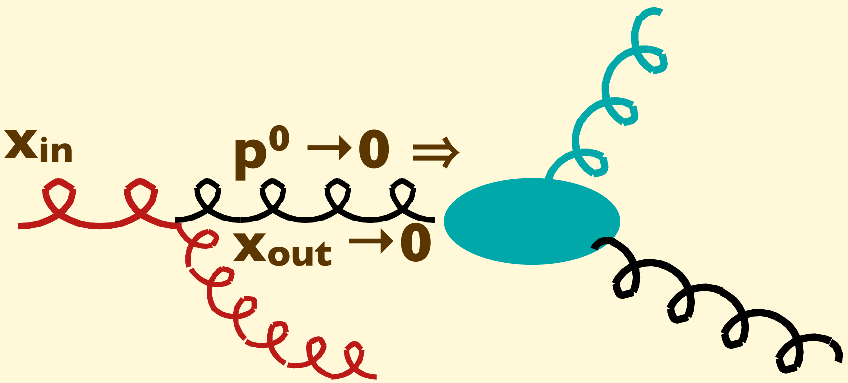


The soft-emission divergence must cancel against the IR divergence of the virtual diagram



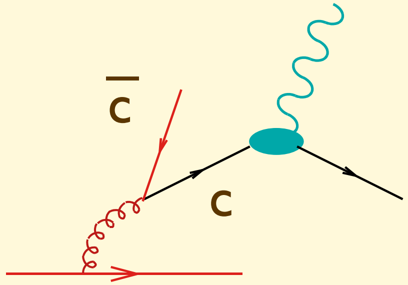
The cancellation cannot take place in the case of collinear divergence, since $\mathbf{x_{out}} \neq \mathbf{x_{in}}$, so virtual and real configurations are not equivalent

Things are different if $\mathbf{p^0 \to 0.}$ In this case, again, $\mathbf{x_{out}} \neq \mathbf{x_{in}}$, no virtual-real cancellation takes place, and an extra singularity due to the $\mathbf{1/p^0}$ pole appears



These are called **small-x** logarithms. They give rise to the double-log growth of the number of gluons at small $\mathbf{x}$ and large $\mathbf{Q}$

Example: charm in the proton



$$\frac{dc(x,Q)}{dt} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} g(y,Q)\, P_{qg}(\frac{x}{y})$$

Assuming a typical behaviour of the gluon density: $g(x,Q) \sim A/x$

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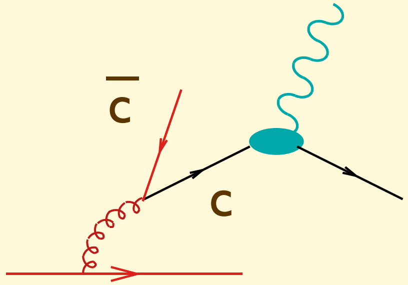

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Assuming a typical behaviour of the gluon density: $g(x,Q) \sim A/x$

and using $P_{qg}(x) = \frac{1}{2}\left[x^2 + (1-x)^2\right]$ we get:

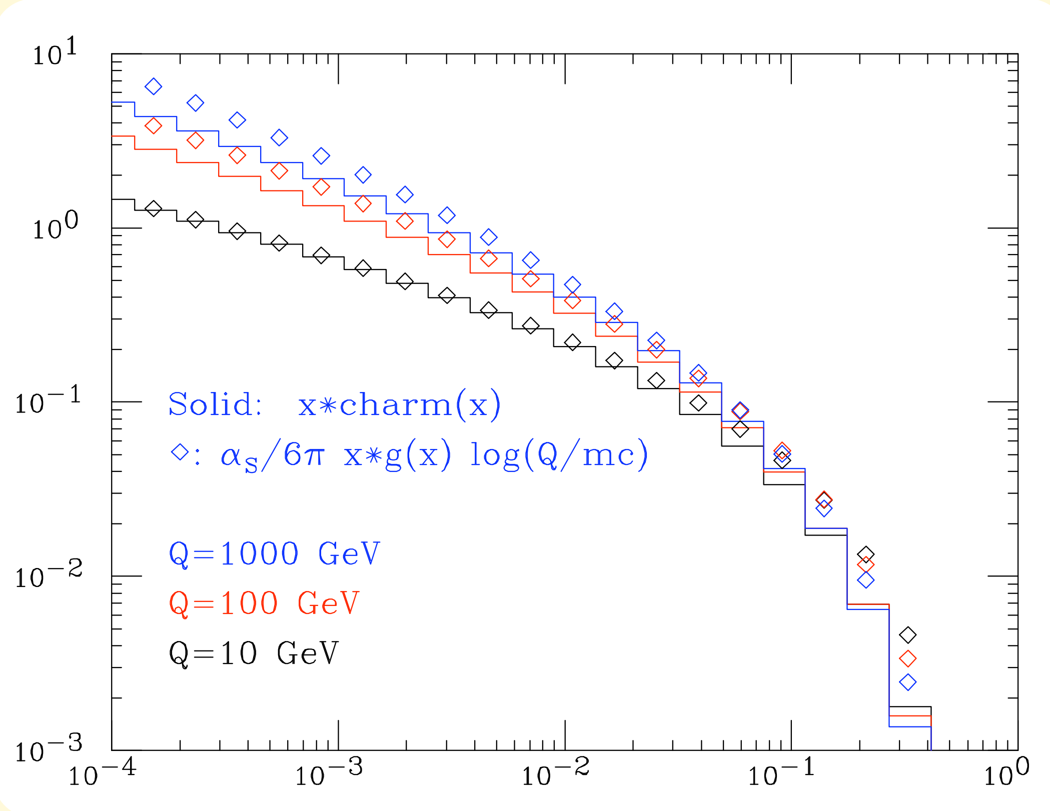
$$\frac{dc(x,Q)}{dt} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} g(x/y,Q)\, P_{qg}(y) = \frac{\alpha_s}{2\pi}\int_x^1 dy \frac{A}{x}\frac{1}{2}[y^2 + (1-y)^2] = \frac{\alpha_s}{6\pi}\frac{A}{x}$$

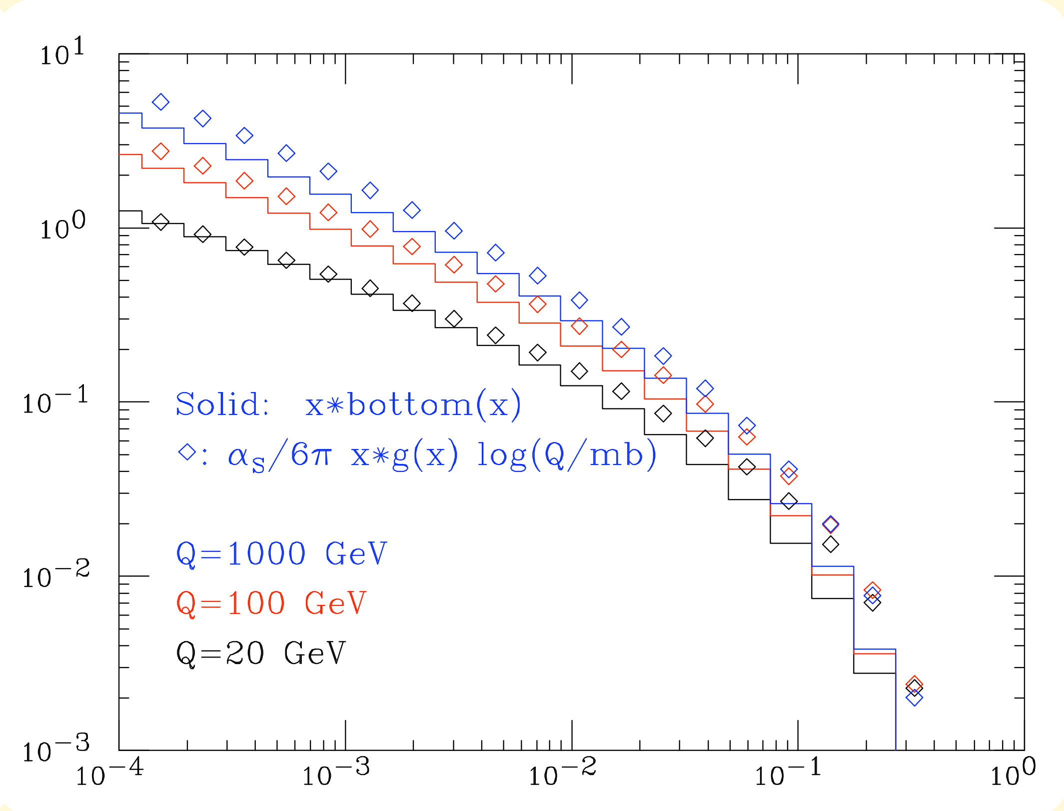
and therefore:

$$\boxed{c(x,Q) \sim \frac{\alpha_s}{6\pi}\log(\frac{Q^2}{m_c^2})\, g(x,Q)}$$

Corrections to this simple formula will arise due to the Q dependence of g(x) and of αs

Numerical example

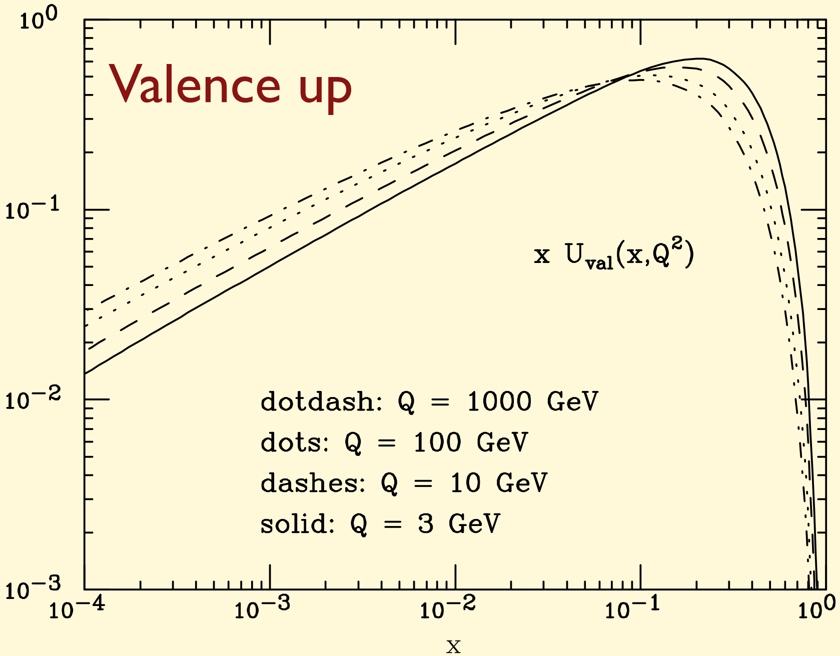


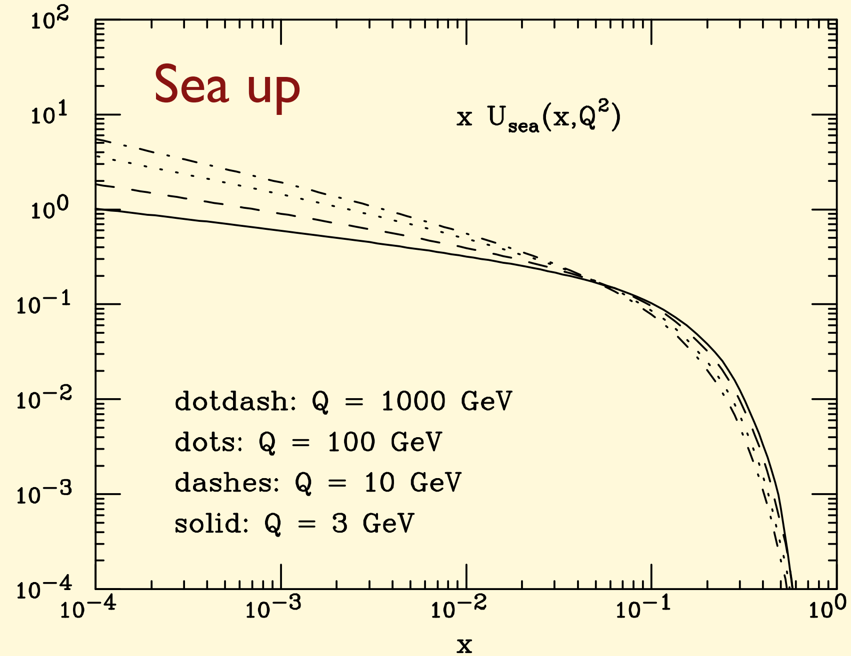


Excellent agreement, given the simplicity of the approximation!

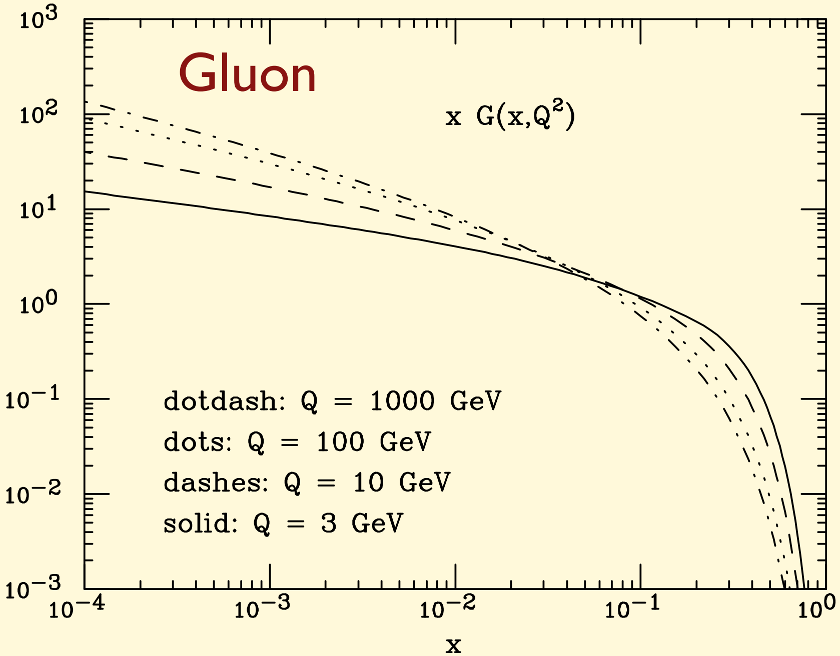
Can be improved by tuning the argument of the log (threshold onset), including a better parameterization of g(x), etc....

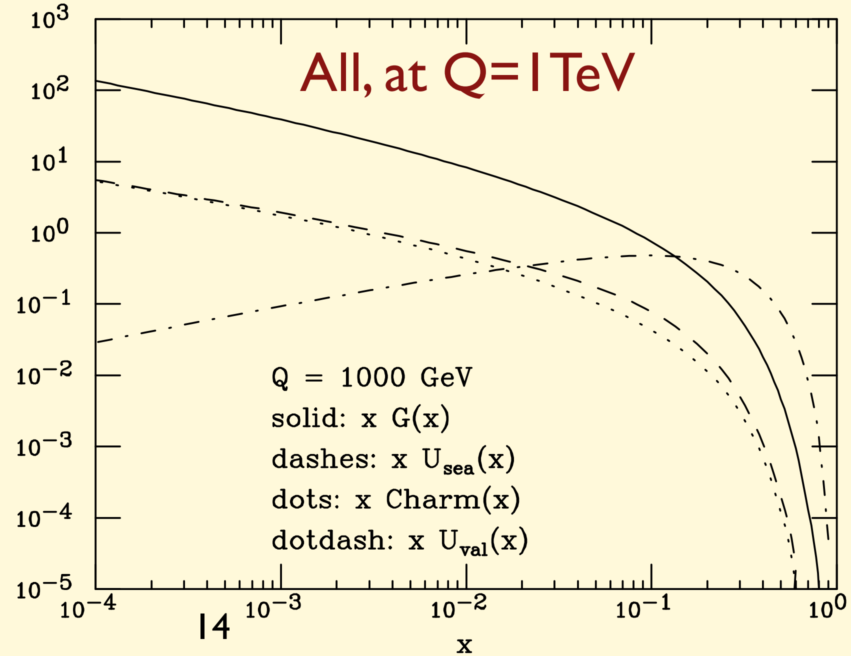
Examples of PDFs and their evolution





Note:
sea ≈10% glue

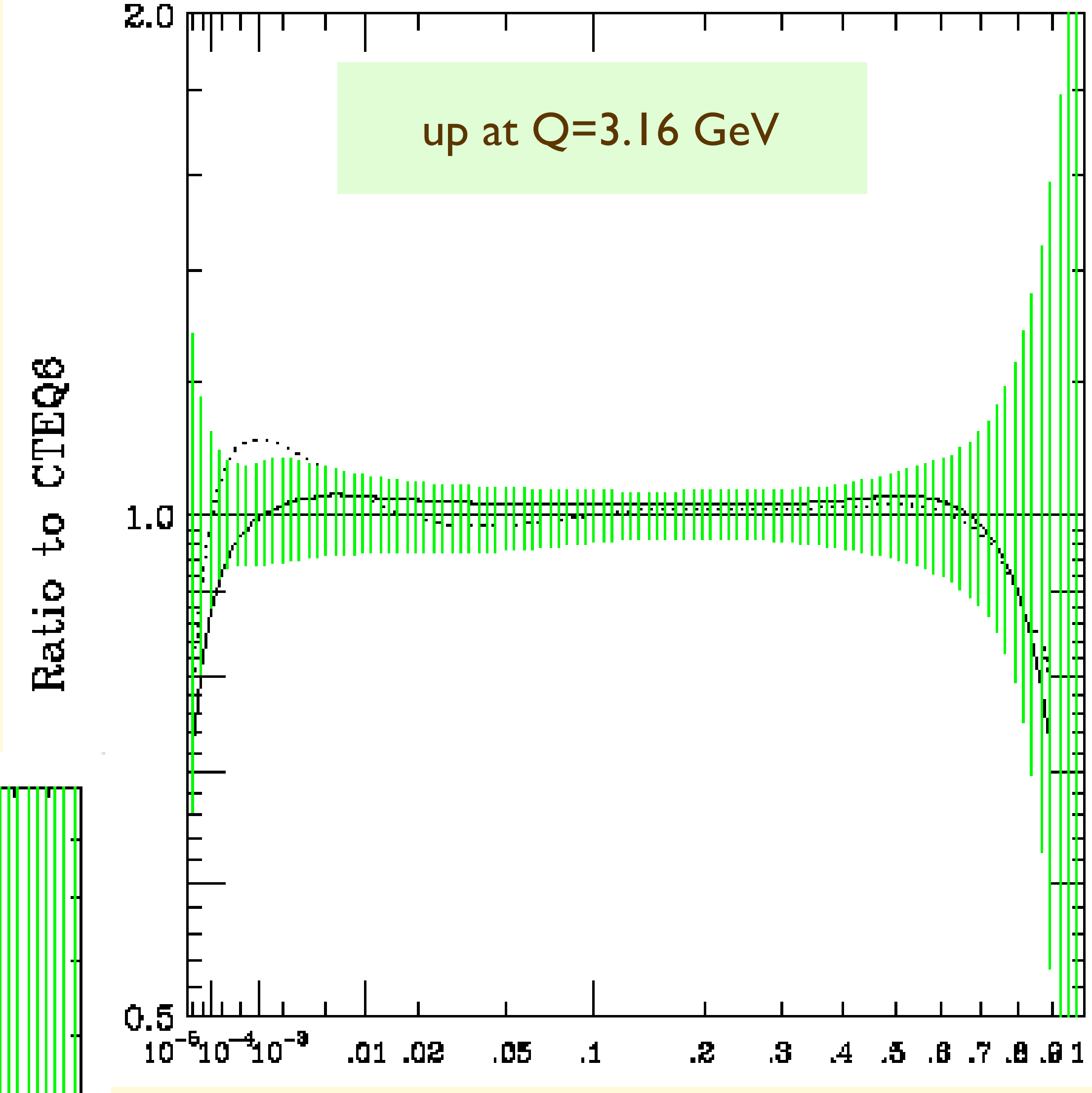


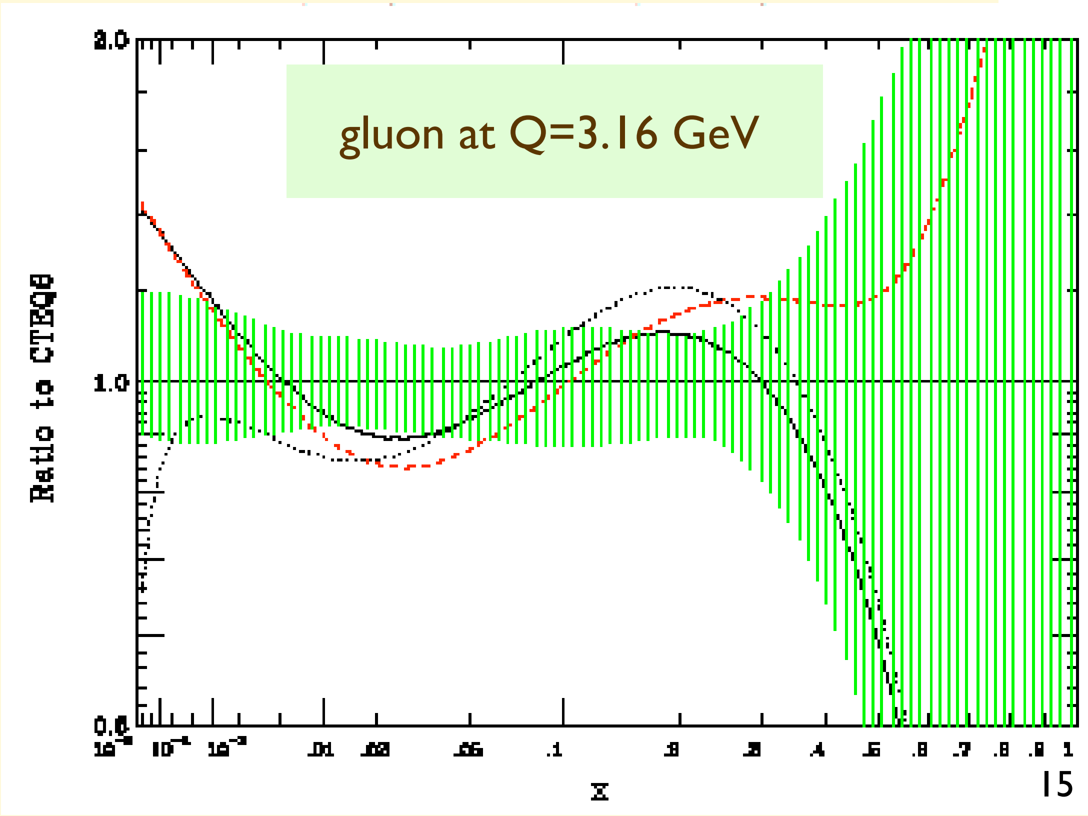


Note:
charm≈up at high Q

PDF uncertainties

Green bands represent the convolution of theoretical and experimental systematics in the determination of PDFs

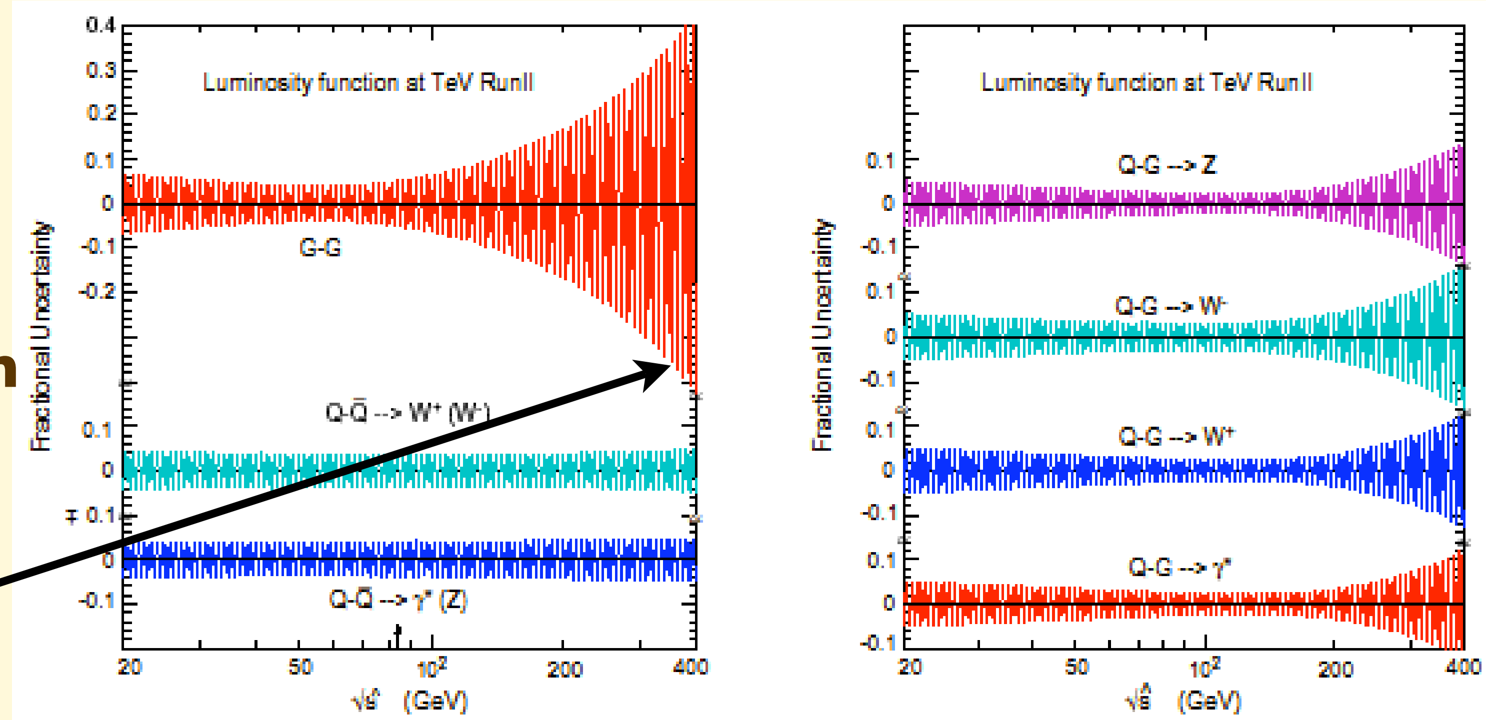




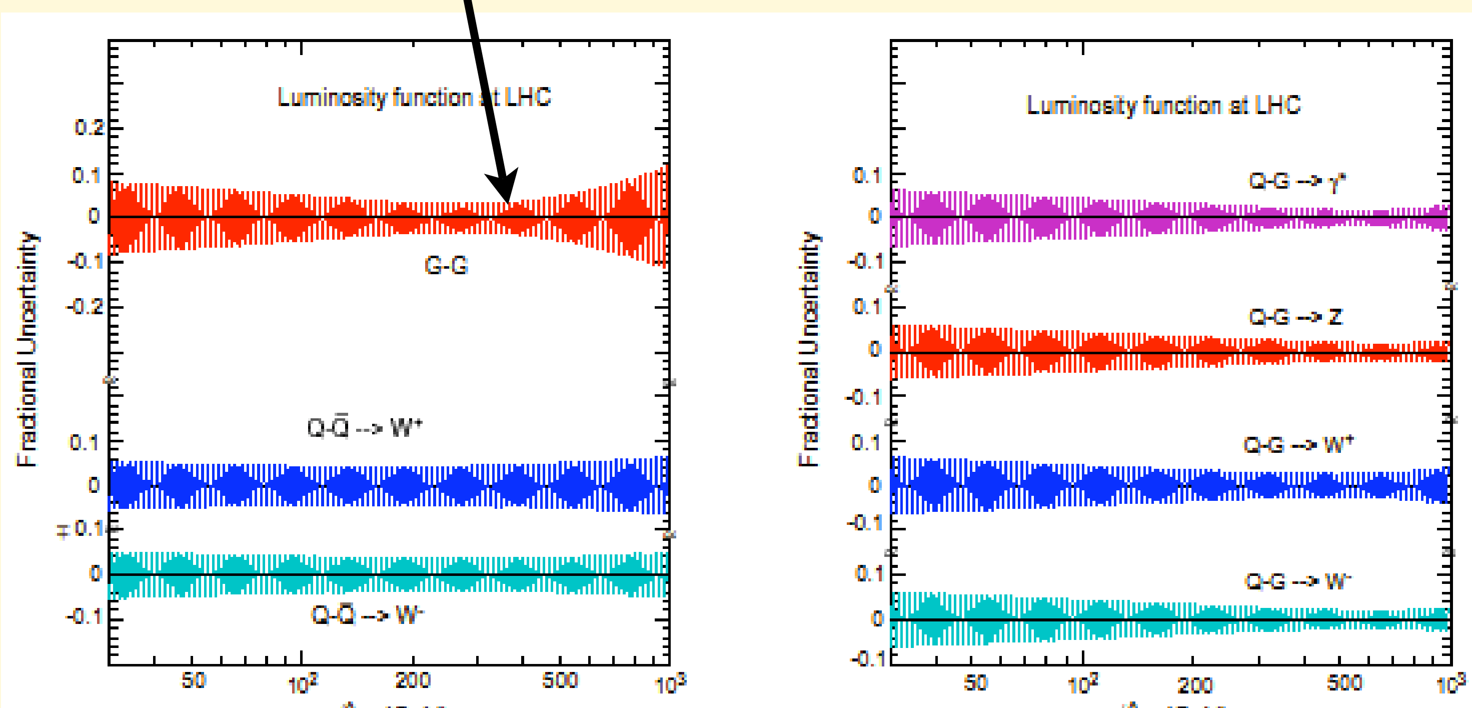
Proton PDFs known to 10-20% for $10^{-3}<x<0.3$, with uncertainties getting smaller at larger Q

PDF luminosity uncertainties

At the Tevatron

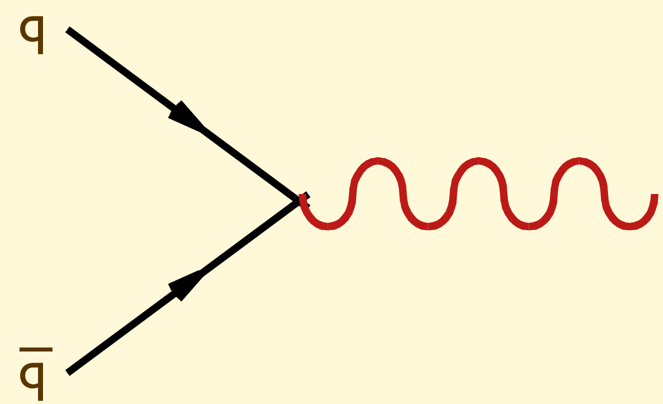


tt production, smaller uncertainty at the LHC!



At the LHC

Example: Drell-Yan processes



$$W \rightarrow \ell\nu$$
$$Z \rightarrow \ell^+\ell^-$$

Properties/Goals of the measurement:

- Clean final state (no hadrons from the hard process)
- Tests of QCD: σ(W,Z) known up to NNLO (2-loops)
- Measure m(W) (➙ constrain m(H))
- constrain PDFs (e.g. $f_{up}(x)/f_{down}(x)$)
- search for new gauge bosons: $q\bar{q} \rightarrow W', Z'$
- Probe contact interactions: $q\bar{q}\ell^+\ell^-$

Some useful relations and definitions

Rapidity: $y = \frac{1}{2} \log \frac{E_W + p^z_W}{E_W - p^z_W}$

Pseudorapidity: $\eta = -\log(\tan \frac{\theta}{2})$

where:

$\tan\theta = \frac{p_T}{p^z}$ and $p_T = \sqrt{p_x^2 + p_y^2}$

Exercise: prove that for a massless particle rapidity=pseudorapidity:

Exercise: using $\tau = \frac{\hat{s}}{S} = x_1 x_2$ and

$$\begin{cases} E_W = (x_1 + x_2)\, E_{\mathrm{beam}} \\ p^z_W = (x_1 - x_2)\, E_{\mathrm{beam}} \end{cases} \Rightarrow y = \frac{1}{2} \log \frac{x_1}{x_2}$$

prove the following relations:

$$x_{1,2} = \sqrt{\tau}\, e^{\pm y} \qquad dx_1\, dx_2 = dy\, d\tau$$

$$dy = \frac{dx_1}{x_1} \qquad d\tau\, \delta(\hat{s} - m_W^2) = \frac{1}{S}$$

LO Cross-section calculation

$$\sigma(pp \to W) = \sum_{q,q'} \int dx_1 dx_2 \, f_q(x_1, Q) \, f_{\bar{q}'}(x_2, Q) \, \frac{1}{2\hat{s}} \int d[PS] \, \overline{\sum_{spin,col}} |M(q\bar{q}' \to W)|^2$$

where:

$$\overline{\sum_{spin,col}} |M(q\bar{q}' \to W)|^2 = \frac{1}{3}\frac{1}{4} 8 g_W^2 |V_{qq'}|^2 \hat{s} = \frac{2}{3} \frac{G_F m_W^2}{\sqrt{2}} |V_{qq'}|^2 \hat{s}$$

$$d[PS] = \frac{d^3 p_W}{(2\pi)^3 p_W^0} (2\pi)^4 \delta^4(P_{in} - p_W)$$

$$= 2\pi \, d^4 p_W \, \delta(p_W^2 - m_W^2) \, \delta^4(P_{in} - p_W) = 2\pi \, \delta(\hat{s} - m_W^2)$$

leading to:

$$\sigma(pp \to W) = \sum_{ij} \frac{\pi A_{ij}}{m_W^2} \, \tau \int_\tau^1 \frac{dx}{x} f_i(x, Q) \, f_j(\frac{\tau}{x}, Q) \equiv \sum_{ij} \frac{\pi A_{ij}}{m_W^2} \, \tau L_{ij}(\tau)$$

where:

$$\frac{\pi A_{u\bar{d}}}{m_W^2} = 6.5\text{nb} \quad \text{and} \quad \tau = \frac{m_W^2}{S}$$

Exercise: Study the function $\tau L(\tau)$

Assume, for example, that $f(x) \sim \frac{1}{x^{1+\delta}}, \quad 0 < \delta < 1$

Then: $$L(\tau) = \int_\tau^1 \frac{dx}{x} \frac{1}{x^{1+\delta}} \left(\frac{x}{\tau}\right)^{1+\delta} = \frac{1}{\tau^{1+\delta}} \log\left(\frac{1}{\tau}\right)$$

and: $$\sigma_W = \sigma_W^0 \left(\frac{S}{m_W^2}\right)^\delta \log\left(\frac{S}{m_W^2}\right)$$

Therefore the W cross-section grows at least logarithmically with the hadronic CM energy. This is a typical behavior of cross-sections for production of fixed-mass objects in hadronic collisions, contrary to the case of e+e- collisions, where cross-sections tend to decrease with CM energy.

Note also the following relation, which allows the measurement of the total width of the W boson from the determination of the leptonic rates of W and Z bosons,

$$\Gamma_W = \frac{N(e^+e^-)}{N(e^\pm\nu)} \left(\frac{\sigma_{W^\pm}}{\sigma_Z}\right) \left(\frac{\Gamma^W_{e\nu}}{\Gamma^Z_{e^+e^-}}\right) \Gamma_Z$$

LHC data

theory

LEP/SLC